# The Analytical Study of Double Diffusive Convection in Visco Elastic Fluids in Porous Medium in an Isotropic Porous Layer

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**ABSTRACT**

In this study, the onset of stationary and oscillatory convection in a horizontal porous layer saturated with viscoelastic fluids heated and soluted from below and observed that the stationary mode is independent of the viscoelastic parameters and specific heat ratio but depends upon the concentration parameters. The important results obtained in their work, include a destabilizing effect of relaxation parameter, Darcy Rayleigh number and Lewis number and a stabilizing effect of solutal Darcy Rayleigh number and retardation parameter.

**Keywords:** Viscoelastic fluids, Porous Layer, Darcy Rayleigh number, Lewis number, Retardation parameter.

## INTRODUCTION

When a fluid saturated porous layer is heated from below, thermal convection occurs due to buoyancy forces in the range exceeding a certain adverse temperature gradient. The convective instability related to Newtonian fluid has been much discussed under Darcy's law **[Horton and Rogers (1945), Lapwood (1948), Katto and Mausko (1967), Cambarnous and Bories (1975), Cheng (1978)]**. It is well known that stationary mode of instability under the principle of exchange of stabilities prevails at the threshold of convective motion in a horizontal porous layer heated from below. But viscoelastic fluid like polymeric solution can exhibit the oscillatory motion caused by overstability.

The problem of Rayleigh Bénard convection in Newtonian fluid has been studied extensively since **Bénard (1990) and Rayleigh (1903**). The Rayleigh Bénard convection in viscoelastic fluids in case of porous medium has been investigated extensively by several researchers and well documented by **Ingham and Pop (1998) Nield and Bejan (1999) and Vafai (2000).** The heat and solute are two diffusive components in thermal convection, also referred to as double diffusive convection. The instability effects in Bénard Rayleigh problems are considered usually at the double diffusive convection induced by thermal and solutal gradients The buoyancy forces can arise not only from density differences due to variations in temperature but also from those due to variations in solute concentration. The problem of double diffusive convection has been extensively investigated for Newtonian as well as non-Newtonian fluids. **Akhatov and Chembarisova (1993**) incorporated the two relaxation parameters into Darcian flow and the analyzed the nonlinear dynamical behavior of thermal convection by using the equations suggested by **Alisaev and Mirzadjanzade (1975).**

The present Chapter is based upon the work of **Tyagi et al. (2012)**, who examined the effect of double diffusion on the onset of oscillatory convection in horizontal porous layer saturated with viscoelastic fluid, previously studied by **Yoon et al. (2004)** for monodiffusion.

## PHYSICAL PROBLEM AND ITS FORMULATION

Consider a viscoelastic fluid saturated porous medium bounded by two infinite horizontal parallel plates situated at a distance 'd' apart. The system is heated and soluted from below in such a way that two boundaries are maintained at a constant temperature difference $\Delta T$ and a constant concentration difference $\Delta C$. The following assumptions have been made in the present analysis:

(i) The saturated fluid is incompressible and non Newtonian.
(ii) The porous medium is isotropic and homogeneous.
(iii) The onset of the thermal and solutal convection is under the Boussinesq's approximation.


https://doi.org/10.5281/zenodo.13268035

(iv) The bottom boundary is kept at temperature $T_1$ , concentration $C_1$ and the upper boundary is kept at lower temperature $T_2$ , and lower concentration $C_2$ with fixed $\Delta T = T_1 - T_2 (> 0)$ and $\Delta C = C_1 - C_2 (> 0)$

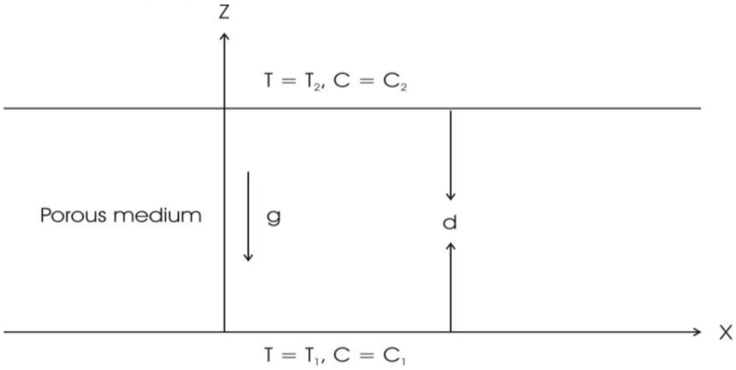


**Fig. 1**

Thus, the above physical system obeys the following set of fundamental equations:

$$\nabla.\mathbf{q} = 0 , \tag{1.1}$$

$$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)q = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)(\nabla p + k_1 \rho g) , \tag{1.2}$$

$$\left(\frac{\partial}{\partial t} + q.\nabla\right)T = \kappa \nabla^2 T , \tag{1.3}$$

$$\left(\frac{\partial}{\partial t} + q.\nabla\right)C = \kappa' \nabla^2 C , \tag{1.4}$$

and $$\rho = \rho_0 [1 - \alpha(T - T_1) + \alpha'(C - C_1)], \tag{1.5}$$

where **q** = (u,v,w) is the velocity vector, K is the permeability of porous medium, $\mu$ is the coefficient of viscosity of the fluid in porous medium, $\bar{\lambda}$ is the relaxation parameter, $\bar{\varepsilon}$ is the retardation parameter, $\rho$ is the density, g is the magnitude of the gravitational acceleration, $\kappa$ is the thermal diffusivity, $\kappa'$ is the solutal diffusivity, $\alpha$ is the thermal expansion coefficient and $\alpha'$ is the solutal expansion coefficient.

**EQUILIBRIUM STATE**

The time independent solution of equations (1.1)-(1.5), called the equilibrium state or basic state of the system, is given by

$$\mathbf{q} = (0,0,0) , \; p = p(z) , \; \rho = \rho(z) , \; T = T(z) \text{ and } C = C(z), \tag{1.6}$$

together with

$$\frac{\partial p}{\partial x} = 0 , \tag{1.6a}$$

$$\frac{\partial p}{\partial y} = 0 , \tag{1.6b}$$


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$$\left(\frac{dp}{dz}+\rho g\right)=0, \qquad (1.6c)$$

$$T-T_1=-\beta z \qquad (1.7)$$

and $$C-C_1=-\beta' z\ . \qquad (1.8)$$

**PERTURBED STATE SOLUTION AND THE LINEARIZED PERTURBATION EQUATIONS**

Suppose that the solution in the basic state is slightly perturbed so that every physical quantity is assumed to be the sum of a mean and a fluctuating component, later designated as primed quantity and assumed to be very small in comparison to its basic state value. The small disturbances are assumed to be the functions of the space as well as time variable.

Hence the perturbed state may be written as:

$$\mathbf{q}=\mathbf{q}+\mathbf{q}'=(0,0,0)+(u',v',w'),\ \ p=p(z)+p',\ T=T(z)+\theta',$$
$$C=C(z)+\gamma' \text{ and } \rho=\rho(z)+\rho'. \qquad (1.9)$$

Substituting equation (1.9) into governing equations (1.1)–(1.5) and linearzing them, we have

**Equation (1.1):**

$$\nabla.\mathbf{q}'=0$$

or $$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0, \qquad (1.10)$$

**Equation (1.2) in x-direction:**

$$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)u' = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial x}(p+p'),$$

on using (1.6a), this leads to

or $$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)u' = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\frac{\partial p'}{\partial x}, \qquad (1.11)$$

**Equation (1.2) in y-direction:**

$$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)v' = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\frac{\partial}{\partial y}(p+p'),$$

on using (1.6b), this leads to

or $$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)v' = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\frac{\partial p'}{\partial y}, \qquad (1.12)$$

**Equation (1.2) in z-direction:**

$$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)w' = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial z}(p+p')+g(\rho+\rho')\right),$$

on using (1.6c), this leads to

or $$\left(1+\bar{\varepsilon}\frac{\partial}{\partial t}\right)w' = -\frac{K}{\mu}\left(1+\bar{\lambda}\frac{\partial}{\partial t}\right)\left(\frac{\partial p'}{\partial z}+\rho' g\right), \qquad (1.13)$$

**Equation (1.3):**

$$\left(\frac{\partial}{\partial t}+q'.\nabla\right)(T+\theta') = \kappa\nabla^2(T+\theta'),$$

or $$\left(\frac{\partial}{\partial t}(T+\theta')+u'\frac{\partial}{\partial x}(T+\theta')+v'\frac{\partial}{\partial y}(T+\theta')+w'\frac{\partial}{\partial z}(T+\theta')\right)T = \kappa\nabla^2((T+\theta'),$$

or $$\frac{\partial\theta'}{\partial t}+w'\frac{dT}{dz}= \kappa\nabla^2\theta',$$

on using (1.7), this leads to

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or $$\frac{\partial\theta'}{\partial t} - \beta w' = \kappa\nabla^2\theta \, , \tag{1.14}$$

where $\beta = \left(\frac{\Delta T}{d}\right)$,

**Equation (1.4):**

$$\left(\frac{\partial}{\partial t} + q'.\nabla\right)(C+\gamma') = \kappa'\nabla^2(C+\gamma') \, ,$$

or $$\left(\frac{\partial}{\partial t}(C+\gamma') + u'\frac{\partial}{\partial x}(C+\gamma') + v'\frac{\partial}{\partial y}(C+\gamma') + w'\frac{\partial}{\partial z}(C+\gamma')\right)T = \kappa'\nabla^2(C+\gamma') \, ,$$

or $$\frac{\partial\gamma'}{\partial t} + w'\left(\frac{dC}{dz}\right) = \kappa'\nabla^2\gamma \, ,$$

or $$\frac{\partial\gamma'}{\partial t} - \beta' w' = \kappa'\nabla^2\gamma \, , \tag{1.15}$$

where $\beta' = \left(\frac{\Delta C}{d}\right)$,

and $$\rho' = \rho_0\left(\beta\theta' - \beta'\gamma'\right), \tag{1.16}$$

where $\mathbf{q}' = (u', v', w')$, $p'$, $\theta'$ and $\gamma'$ are disturbances in velocity, pressure, temperature and concentration.

**NORMAL MODE ANAYSIS AND THE EIGEN VALUE PROBLEM**

Under the normal mode analysis, we assume the time-dependent periodic disturbances of the form

$$[w', p', \theta' \; \gamma'] = [w(z), p(z), \theta(z), \gamma(z)]e^{\left(ia_x x + ia_y y + nt\right)} \, , \tag{1.17}$$

where $a_x$ and $a_y$ are the real wave numbers in x and y directions respectively, and $n = (n_r + in_i)$ in general, is a complex number.

Substituting (1.17) in equations (1.10)-(1.16), the stability governing equations of the system are given by

$$Dw = -\left(ia_x u + ia_y v\right), \tag{1.18}$$

$$\left(1+\bar{\varepsilon}\, n\right)u = -\frac{K}{\mu}\left(1+\bar{\lambda}\, n\right)ia_x p \, , \tag{1.19}$$

$$\left(1+\bar{\varepsilon}\, n\right)v = -\frac{K}{\mu}\left(1+\bar{\lambda}\, n\right)ia_y p \, , \tag{1.20}$$

$$\left(1+\bar{\varepsilon}\, n\right)w = -\frac{K}{\mu}\left(1+\bar{\lambda}\, n\right)\left[\frac{dp}{dz} - \rho_0\left(\alpha\theta - \alpha'\gamma\right)g\right], \tag{1.21}$$

$$\theta\, n - \left(\frac{\Delta T}{d}\right)w = \kappa\left(D^2 - a^2\right)\theta \, , \tag{1.22}$$

$$\gamma\, n - \left(\frac{\Delta C}{d}\right)w = \kappa'\left(D^2 - a^2\right)\gamma \, , \tag{1.23}$$

where $D \equiv \frac{d}{dz}$ and $a = \sqrt{a_x^2 + a_y^2}$ .

And after eliminating various physical quantities in favour of w from equations (1.18)-(1.21), we get

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$$\left(1+\bar{\varepsilon}\,n\right)\left(D^2-a^2\right)w=-\frac{K\rho_0 g}{\mu}\left(1+\bar{\lambda}\,n\right)\left(\alpha\theta-\alpha'\gamma\right)a^2. \tag{1.24}$$

Now we non-dimensionalize equations (1.24), (1.22) and (1.23) by introducing the following non-dimensional parameter

$$\left.\begin{array}{l}\left(x*,y*,z*\right)=\left(\frac{x}{d},\frac{y}{d},\frac{z}{d}\right),\; a=\frac{k}{\left(1/d\right)},\; \sigma=\frac{nd^2}{\kappa},\; D^*=\frac{D}{\left(1/d\right)},\\ \text{and } \theta^*=\frac{\theta\kappa}{\beta d^2 U_0},\; w^*=\frac{w}{U_0},\; \gamma^*=\frac{\gamma\kappa'}{\beta' d^2 U_0}\end{array}\right\}, \tag{1.25}$$

The final non-dimensional equations, after dropping the stars, are obtained as

$$\left(1+\varepsilon\sigma\right)\left(D^2-a^2\right)w=-\left(1+\lambda\sigma\right)\left(\frac{K\rho_0 g\alpha}{\mu}\theta-\frac{K\rho_0 g\alpha'}{\mu}\gamma\right)a^2, \tag{1.26}$$

$$\left(D^2-a^2-\sigma\right)\theta=-\frac{(\Delta T)d}{\kappa}w, \tag{1.27}$$

and

$$\left(D^2-a^2-\sigma Le\right)\gamma=-\frac{(\Delta C)d}{\kappa'}w. \tag{1.28}$$

After eliminating $\theta$ and $\gamma$ from equations (1.26)-(1.28), the combined stability governing equation is obtained as

$$\begin{aligned}&\left(1+\varepsilon\sigma\right)\left[\left(D^2-a^2\right)^3-\sigma\left(1+Le\right)\left(D^2-a^2\right)^2+Le\,\sigma^2\left(D^2-a^2\right)\right]w\\ &\quad=a^2\left(1+\lambda\sigma\right)\left[\left(Ra_D-Rs_D\right)\left(D^2-a^2\right)w-\sigma\left(Ra_D\,Le-Rs_D\right)w\right],\end{aligned} \tag{1.29}$$

where $Le=\frac{\kappa}{\kappa'}$ is Lewis number, $Ra_D=\frac{Kg\alpha\Delta T\,d}{\kappa\nu}$ is Darcy Rayleigh number, $Rs_D=\frac{Kg\alpha'\Delta C\,d}{\kappa'\nu}$ is solutal Darcy Rayleigh number, $\lambda=\frac{\kappa}{d^2}\bar{\lambda}$ is non-dimensional relaxation parameter, $\varepsilon=\frac{\kappa}{d^2}\bar{\varepsilon}$ is non-dimensional retardation parameter and $\nu=\frac{\mu}{\rho_0}$ is kinematic viscosity.

The appropriate boundary conditions are

$$w=D^2w=D^4w=0 \quad \text{at } z=0,1. \tag{1.30}$$

The eigen value problem given by equation (1.29) under the boundary conditions (1.29) involving $Ra_D$, $Rs_D$, a, $\varepsilon$, $\lambda$, Le and $\sigma$ as parameters, is solved upon assuming that the amplitude w(z) is small enough and can be expressed as

$$w=w_0\sin\left(m\pi z\right) \text{ for } m=1,2,3\ldots\ldots \tag{1.31}$$

Substituting equation (1.31) with equation (1.31) into equation (1.29), we obtain

$$A_1\sigma^3+A_2\sigma^2+A_3\sigma+A_4=0, \tag{1.32}$$

where

$$A_1=\varepsilon Le\left(m^2\pi^2+a^2\right), \tag{1.33}$$

$$A_2=\varepsilon\left(1+Le\right)\left(m^2\pi^2+a^2\right)^2+Le\left(m^2\pi^2+a^2\right)-\lambda a^2\left(Ra_D Le-Rs_D\right), \tag{1.34}$$

$$\begin{aligned}A_3=&\,\varepsilon\left(m^2\pi^2+a^2\right)^3+\left(1+Le\right)\left(m^2\pi^2+a^2\right)^2\\ &-\lambda a^2\left(Ra_D-Rs_D\right)\left(m^2\pi^2+a^2\right)-a^2\left(Ra_D\,Le-Rs_D\right),\end{aligned} \tag{1.35}$$

and

$$A_4=\left(m^2\pi^2+a^2\right)^3-a^2\left(Ra_D-Rs_D\right)\left(m^2\pi^2+a^2\right). \tag{1.36}$$

We observe that $A_1$ is positive definite whereas $A_2$, $A_3$ and $A_4$ can be positive or negative depending upon the physical parameters.

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**STATIONARY CONVECTION**

For stationary convection at marginal state $(\sigma_r = 0, \sigma_i = 0)$, $Ra_D$ is given by

$$Ra_D = Rs_D + \frac{\left(m^2\pi^2 + a^2\right)^2}{a^2}. \tag{1.37}$$

Minimization of $Ra_D$ with respect to wave number 'a' yields the critical Darcy Rayleigh number for stationary convection, for lowest mode i.e. m = 1, as

$$Ra_{D,min}^{stat.} = Rs_D + 4\pi^2, \tag{1.38}$$

and the corresponding critical wave number is $\pi$.

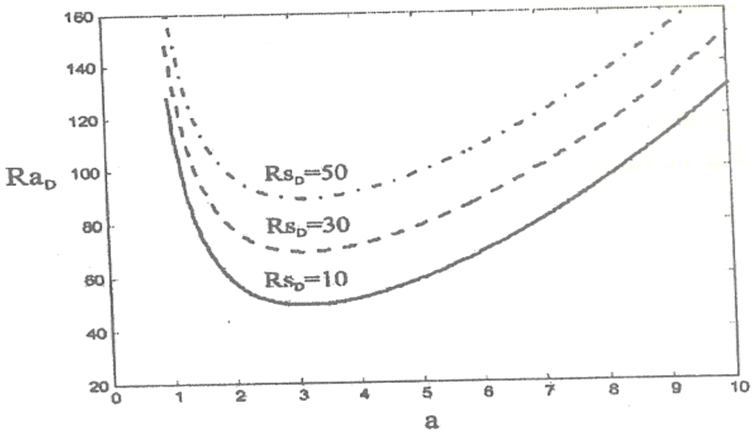


**Fig. 2: Variations of critical Darcy Rayleigh number ($Ra_D$) with wave number (a) for different value of solutal Darcy Rayleigh number ($Rs_D$).**

It is well known result for stationary convection at $\sigma$ = 0 previously discussed by **Horton and Rogers (1945)**, **Lapwood (1948)**, **Combarnous and Bories (1975)**, **Cheng (1978) and Yoon et al. (2004)**.

The variations of critical Darcy Rayleigh number with wave number for $Rs_D$ = 10, 30 and 50 is shown by Fig. 2. It shows that the stabilizing effect of solutal Darcy Rayleigh number.

**ANALYTICAL DISCUSSION**

If $\sigma_1, \sigma_2$ and $\sigma_3$ are the roots of the cubic equation (1.32), then

$$\sigma_1 + \sigma_2 + \sigma_3 = -\frac{A_2}{A_1}, \tag{1.39}$$

$$\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 = \frac{A_3}{A_1} \tag{1.40}$$

and $$\sigma_1\sigma_2\sigma_3 = -\frac{A_4}{A_1}. \tag{1.41}$$

The following theorems are describing the nature of the roots of equation (1.32).

**Theorem 1:** If $A_4 < 0$, then the system is unstable.

**Proof:** When $A_4 < 0$, the product of the roots

$$\sigma_1\,\sigma_2\,\sigma_3 = \frac{A_4}{A_1},$$

it is clear from above equation that if all roots are real, then either only one root is positive or all three roots are positive. This ensures the instability of the system.

Further, if all three roots are negative or one root is real and negative and the other two roots are complex with either positive or negative real parts, equation (1.41) is contradicted. Thus, if $A_4$<0, either all roots are positive or exactly one root is positive and the other two roots may be either complex or both negative. In all situations the instability of the system is established.

The proof of theorem is complete.

**Theorem 2:** The unstable modes which exist under the condition $A_4 < 0$ are non-oscillatory for $A_2 > 0$.

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**Proof:** Multiplying equation (1.32) by $\sigma^*(=\sigma_r - i\sigma_i)$, the complex conjugate of $\sigma\left(=\sigma_r + i\sigma_i\right)$, we get

$$A_1\sigma^3\sigma^* + A_2\sigma^2\sigma^* + A_3\sigma\sigma^* + A_4\sigma^* = 0$$

or $$A_1\sigma^2|\sigma|^2 + A_2\sigma|\sigma|^2 + A_3|\sigma|^2 + A_4\sigma^* = 0$$

or $$A_1(\sigma_r + i\sigma_i)^2|\sigma|^2 + A_2(\sigma_r + i\sigma_i)|\sigma|^2 + A_3|\sigma|^2 + A_4(\sigma_r - i\sigma_i) = 0, \quad (1.42)$$

on equating the imaginary part of equation (1.42) equal to zero, we have

$$\sigma_i\left[(2A_1\sigma_r + A_2)|\sigma|^2 - A_4\right] = 0. \quad (1.43)$$

Now when $A_2>0$ in addition to condition $A_4<0$, then the quantity inside the brackets in equation (1.43) is positive definite and so we necessarily have

$$\sigma_i = 0.$$

Therefore, the unstable modes which exist are non-oscillatory under the condition $A_4 < 0$ and $A_2 > 0$.

The proof of theorem is complete.

**Theorem 3:** If $A_2<0$ and $A_4<0$, then the bounds on $\sigma_r$ for oscillatory unstable modes, if exist, are given by

$$0 < \sigma_r < \frac{|A_2|}{2A_1}.$$

**Proof:** If $A_2 < 0$ and $A_4 < 0$, then equation (1.43) can be written as

$$\sigma_i\left[(2A_1\sigma_r - |A_2|)|\sigma|^2 + |A_4|\right] = 0, \quad (1.44)$$

when the oscillatory unstable modes $(\sigma_i \neq 0, \sigma_r > 0)$ exist, then, for consistency of equation (1.44) necessarily requires the following bounds on $\sigma_r$:

$$(2A_1\sigma_r - |A_2|) < 0 \Rightarrow 0 < \sigma_r < \frac{|A_2|}{2A_1}. \quad (1.45)$$

The proof of theorem is complete.

**Theorem 4:** The oscillatory unstable modes, if exist under the conditions $A_2<0$ and $A_4 < 0$, lie outside the circle

$$|\sigma|^2 > \frac{|A_4|}{|A_2|}.$$

**Proof:** If $A_2 < 0$ and $A_4 < 0$, then equation (1.43) can also be written as

$$\sigma_i\left[2A_1\sigma_r + \left(|A_4| - |A_2||\sigma|^2\right)\right] = 0, \quad (1.46)$$

for consistency of equation (1.46), when the oscillatory unstable modes $(\sigma_i \neq 0, \sigma_r > 0)$ exist, necessarily requires

$$\left(|A_4| - |A_2||\sigma|^2\right) < 0,$$

provides the following bounds on $\sigma_r$:

$$|\sigma|^2 > \frac{|A_4|}{|A_2|}. \quad (1.47)$$

The proof of theorem is complete.

**Theorem 5:** If $A_3 < 0$ and $A_4 < 0$, then the unstable modes which exist are non-oscillatory.

**Proof:** Dividing equation (1.32) by $\sigma$, we get

$$A_1\sigma^2 + A_2\sigma + A_3 + \frac{A_4}{\sigma} = 0,$$

on multiplying by $\sigma^*$ complex conjugate of $\sigma$, in above equation we get

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$$A_1\ |\sigma|^2\sigma + A_2|\sigma|^2 + A_3\sigma* + A_4\frac{\sigma*}{|\sigma|^2} = 0\,,$$

now equating imaginary part of the above equation equal to zero, we get

$$\sigma_i\left(A_1|\sigma|^2 - A_3 - \frac{2A_4}{|\sigma|^2}\sigma_r\right) = 0\,, \tag{1.48}$$

If $A_3<0$ and $A_4<0$, then equation (1.48) can be written as

$$\sigma_i\left(A_1|\sigma|^2 + |A_3| + \frac{2|A_4|}{|\sigma|^2}\sigma_r\right) = 0\,, \tag{1.49}$$

now for consistency of equation (1.49), when the unstable modes $(\sigma_r > 0)$ exist, necessarily requires that

$$\sigma_i = 0\,.$$

It shows that unstable modes are non-oscillatory.

The proof of theorem is complete.

## Numerical Results and Discussion

For given wave numbers and physical parameters, roots of equation (1.32) have been obtained and the range of stable and unstable wave numbers for a chosen parameter is shown graphically.

For fixed $\varepsilon$ (=0.25), $Le$ (=0.25), $\lambda$ (=0.4) and $Rs_D$ (=10), the range between critical wave numbers $a_{c1}$ and $a_{c2}$ increases as Darcy Rayleigh number $Ra_D$ increases (**Fig. 3**). This shows that a destabilizing character of Darcy Rayleigh number.

For fixed $\varepsilon$ (=0.25), $Le$ (=0.25), $\lambda$ (=0.4) and $Ra_D$ (=100), the range between critical wave numbers $a_{c1}$ and $a_{c2}$ decreases as solutal Darcy Rayleigh number $Rs_D$ increases (**Fig. 4**). This proves a stabilizing character of the solutal Darcy Rayleigh number.

For fixed $\lambda$ (=1.25), $Le$ (=0.25), $Ra_D$ (=50) and $Rs_D$ (=10), the range between critical wave numbers $a_{c1}$ and $a_{c2}$ decreases as retardation parameter $\varepsilon$ increases (**Fig. 5**). It is clear from Fig. 5 that as the retardation parameter $\varepsilon$ increases, the range of unstable wave numbers decreases. This shows that the retardation parameter $\varepsilon$ has stabilizing effect.

For fixed $\varepsilon$ (=0.25), $Le$ (=0.25), $Ra_D$ (=50) and $Rs_D$ (=10), the range between critical wave numbers $a_{c1}$ and $a_{c2}$ increases as relaxation parameter $\lambda$ increases (**Fig. 6**). Therefore, the relaxation parameter $\lambda$ has destabilizing effect.

For fixed $\varepsilon$ (=0.25), $\lambda$ (=0.4), $Ra_D$ (=50) and $Rs_D$ (=10), the range between critical wave numbers $a_{c1}$ and $a_{c2}$ increases as Lewis number $Le$ increases (**Fig. 7**). This establishes a destabilizing role of Lewis number Le.

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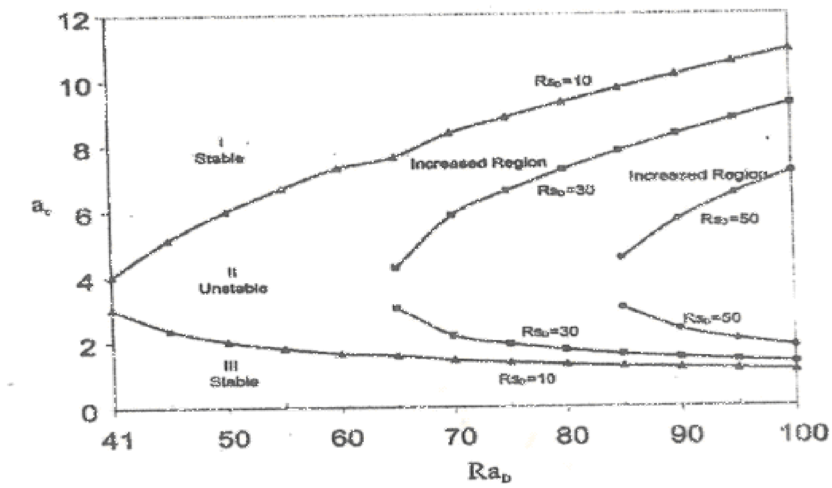


**Fig. 3: Variations of critical wave number $a_c$ with $Ra_D$ for different values of $Rs_D$.**

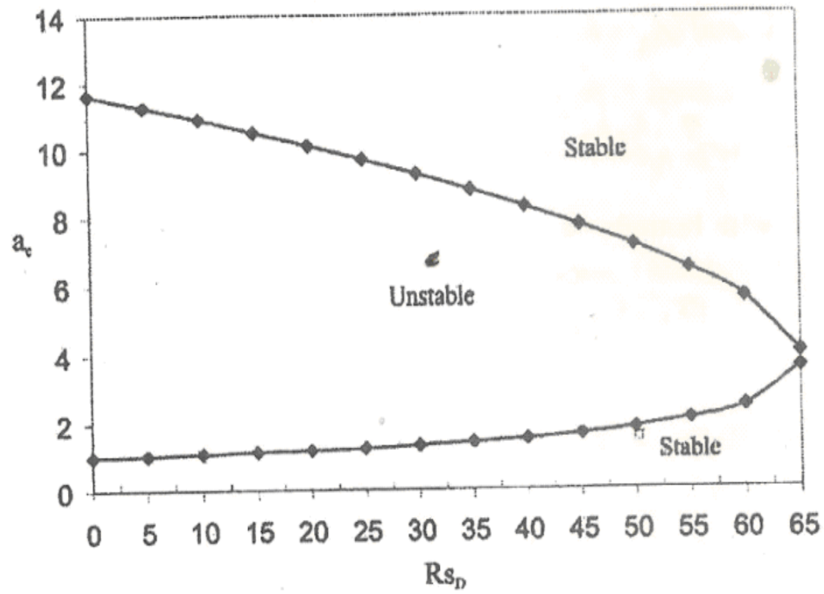


**Fig. 4: Variations of critical wave number $a_c$ with $Rs_D$.**

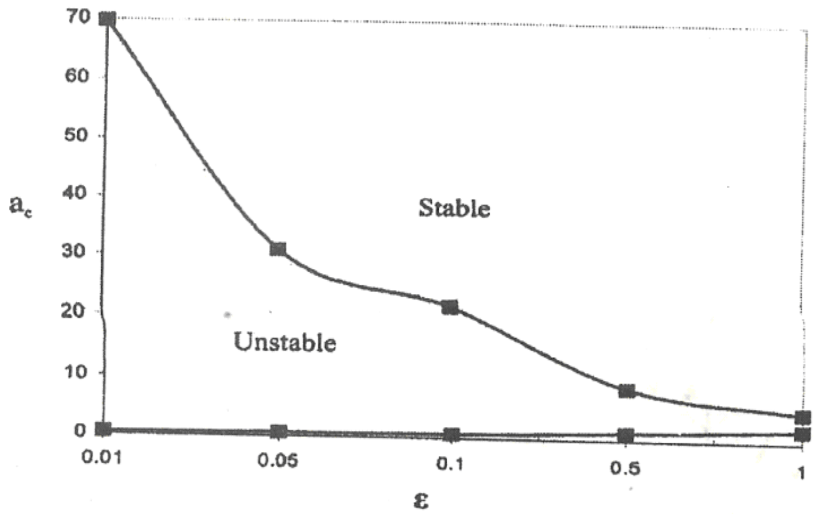


**Fig. 5: Variations of critical wave number $a_c$ with $\varepsilon$ .**


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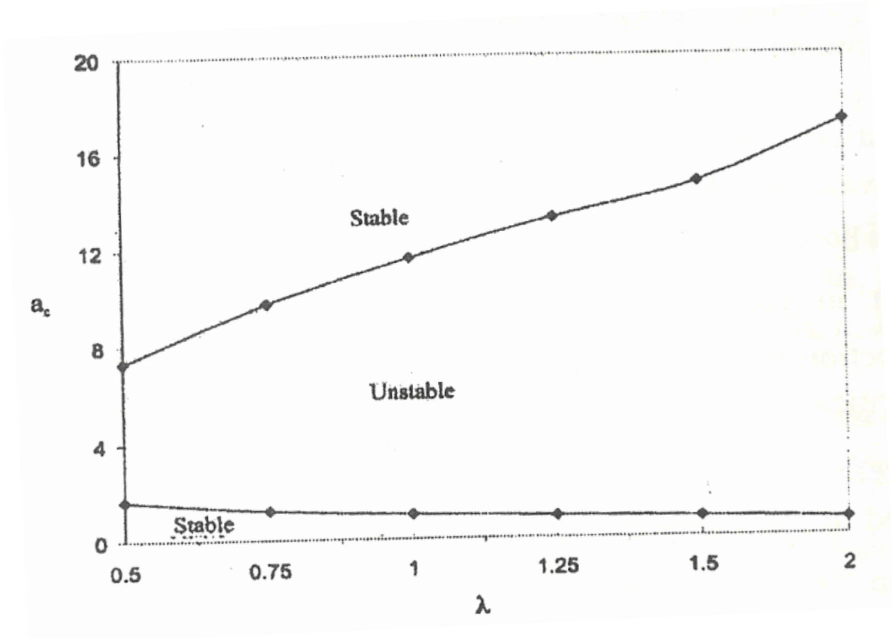


**Fig. 6: Variations of critical wave number $a_c$ with $\lambda$ .**

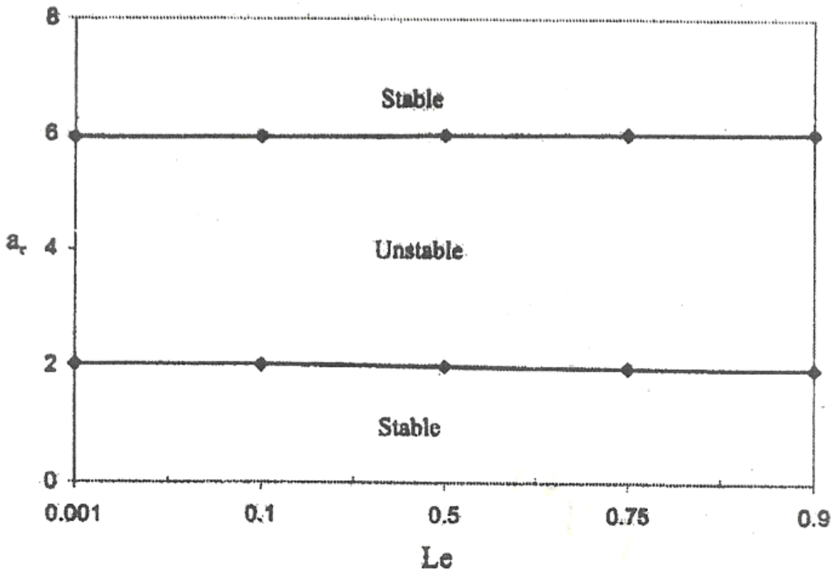


**Fig. 7: Variations of critical wave number $a_c$ with Le**.

## CONCLUDING REMARK

This Chapter is based upon the work of the "**Onset of stationary and oscillatory convection in a horizontal porous layer saturated with viscoelastic fluids heated and soluted from below"** and observed that the stationary mode is independent of the viscoelastic parameters and specific heat ratio but depends upon the concentration parameters. The important results obtained in their work, include a destabilizing effect of relaxation parameter, Darcy Rayleigh number and Lewis number and a stabilizing effect of solutal Darcy Rayleigh number and retardation parameter.

## LIST OF SYMBOLS

| | |
|---|---|
| a | Wave number |
| $\alpha$ | Thermal expansion coefficient |
| $\alpha'$ | Solute expansion coefficient |


https://doi.org/10.5281/zenodo.13268035

| | |
|---|---|
| β | $\left(=\frac{\Delta T}{d}\right)$ |
| β' | $\left(=\frac{\Delta C}{d}\right)$ |
| C | Solute concentration |
| D | Height of the fluid layer |
| K | Permeability |
| $\mathbf{k}_1$ | (0, 0, 1) |
| **K** | Permeability tensor $\left[= k_x\left(\hat{i}\hat{i}+\hat{j}\hat{j}\right)+k_z\left(\hat{k}\hat{k}\right)\right]$ |
| $K_1$ | $=\frac{k_x}{k_z}$ |
| Le | Lewis number $\left(=\frac{\kappa}{\kappa'}\right)$ |
| N | Buoyancy ratio $\left(=\frac{\alpha'\beta'}{\alpha\beta}\right)$ |
| p | Pressure |
| **q** | Velocity (u, v, w) |
| $Ra_D$ | Darcy Rayleigh number $\left(=\frac{Kg\alpha\beta d^2}{kv}\right)$ |
| $Rs_D$ | Solutal Rayleigh number $\left(=\frac{Kg\alpha'\beta' d^2}{k'v}\right)$ |
| T | Temperaturte |
| t | Time |
| x, y, z | Space coordinates |

## Greek Symbols

| | |
|---|---|
| $\bar{\varepsilon}$ | Retardation time |
| ε | Dimensionless retardation time $\left(=\frac{k}{d^2}\bar{\varepsilon}\right)$ |
| κ | Thermal diffusivity |
| κ' | Solutal diffusivity |
| $\bar{\lambda}$ | Relaxation time |
| λ | Dimensionless Relaxation time $\left(=\frac{k}{d^2}\bar{\lambda}\right)$ |
| ν | Kinematic viscosity $\left(=\frac{\mu}{\rho_0}\right)$ |
| μ | Viscosity |
| ρ | Density |

## Subscripts

| | |
|---|---|
| c | Critical |
| 0 | References value |


https://doi.org/10.5281/zenodo.13268035

min. Minimum

## Superscripts

| | |
|---|---|
| * | Dimensionless quantity |
| ' | Perturbed quantities |
| stat. | Stationary |
| osc. | Oscillatory |

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