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THE DOMINANCE OF ANNIHILATOR IN PRODUCT GRAPHS

K. V. Suryanarayana Rao[1*], S. Venkateswarlu[1], D. Dastagiri Babu[1], B. Siva Kumar Reddy[2]

[1] Department of Mathematics, Rajeev Gandhi Memorial College of Engineering and Technology, Nandyal, Andhra Pradesh, India-518501

[2] Department of Mathematics, Guru Nanak Institute of Technology (Autonomous), Ibrahimpatnam, Ranga Reddy District, Telangana, India-501506

[*]Corresponding Author: K. V. Suryanarayana Rao

Abstract

The theory of graph dominance is discussed in the work. Kulli and Janakiram [4] showed that the split domination in graphs. The Annihilator dominating set and Annihilator dominating number are new type of parameters on dominance that we describe in this work. We also took a look at some of the features of the Annihilator dominating number of product graphs and found some interesting findings.

Key words: Annihilator domination set, Product Graphs, Annihilator domination number, Domination.

1. Introduction

A dynamic subfield of modern mathematics, Graph theory has grown rapidly in the last forty years. Its wide range of applications in discrete optimization, combinatorial problems, and classical algebraic challenges has made it a vital field of study. Graph theory has an impact on engineering, linguistics, physical sciences, social sciences, and biology. Lately, Graph theory research has become more and more focused on the notion of domination. This area of study has grown because of Graph theory's adaptability and its relationship to NP-completeness problems, which has encouraged more research into related complex problems.

Although chronologically stemmed from de Jaenisch's (1862) concern regarding queens on a chessboard, the topic of domination in networks began to be extensively studied in Graph theory approximately 1960. The idea of the identity of a Graph's dominance number was initially put

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forward by Berge (1958) beneath the name "Coefficient of External Stability". Moreover, in 1962, Ore termed it the pair of "Dominating set" and the "Dominating number". The relevance of dominant sets became apparent in a comprehensive study from Cockkayne and Hedetneiemi (1977), and subsequent to that, the acronym $\gamma(G)$ has been utilized extensively employed to indicate a Graph's dominance number.

In the two decades that have passed since Haynes [2] survey, the field has been a significant attention, as seen by the publication over 1200 articles. Many academics, including Ore, Harary, Konig, Bauer, Berge, Lasker, Alavi, Hedetniemi, Cockayne, Chartrand, Allan, Walikar, Sampath, Acharya, Armugam, Vangipuram, Nagaraja Rao, Neeralgi have made significant contributions to this enormous body of work. The study they conducted on dominance numbers and associated subjects has contributed to continuous progress throughout this field. New publications, particularly a book on dominance, have stimulated additional study and categorized a large amount of literature into functional subfields.

In graphs, the notion of split domination have been presented by Kulli & Janakiram [4], according to them, the split dominating set as well as split domination number and investigated their connections to other parameters such as connected dominance number and dominance number. We now know more about these ideas because to Sampath [5] worked on a few domination parameters of a graph and Suryanarayana Rao & Sreenivasan [6, 7] on the dominating parameters of arithmetic and product graphs. Sharma and Sharma [8] investigated annihilator domination number of tensor product of path graphs. Aparna et al. [9, 10] studied the Split and Annihilator Dominance of some strong product Graphs and Fuzzy Graphs. Expanding upon these notions, we provide the notion of the Annihilator Dominance Set and its corresponding Annihilator Dominance Number, investigating its consequences within the framework of product graphs. The expressions and information demonstrated in this paper are similar to [3] and [1].

Important definitions:

1.1: Dominating set: If every vertex in W \ M is adjacent to a vertex in M, then a subset M of W is a dominating set of H.

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1.2: Dominating number: A dominating set of minimum cardinality is its dominating number γ(H) of H.

1.3: Split dominating set : If the induced sub graph <W-M> is disconnected, then dominating set M of graph H is known to be a split dominating set.

1.4: Split domination number: The dominating number $\gamma_s(H)$ of H is the split dominating set of minimum cardinality.

1.5: Kronecker Product of two graphs

The Kronecker product of two simple graphs H_1 and H_2 with their vertex sets $W_1 : \{u_1, u_2, ...\}$ *and* $W_2 : \{v_1, v_2, ...\}$ respectively is defined as a graph having vertex set as $W_1 \times W_2$ and its vertices (u_i, v_j), (u_k, v_l) are adjacent iff $u_i u_k$ and $v_j v_l$ are edges in H_1 and H_2 respectively.

The symbol for this product graph is H_1 (K) H_2.

1.6: Cartesian product of two graphs

The Cartesian product of two simple graphs H_1 and H_2 with their vertex sets $W_1 : \{u_1, u_2, ...\}$ *and* $W_2 : \{v_1, v_2, ...\}$ respectively, is defined as a graph with vertex set as $W_1 \times W_2 : \{w_1, w_2, ...\}$ and two vertices $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are adjacent, if and only if either $(i) u_1 = u_2$ *and* $v_1 v_2 \in E(H_2)$ *or* (ii) $u_1 u_2 \in E(H_1)$ *and* $v_1 = v_2$.

This product graph is denoted by H_1 (C) H_2.

1.7: Lexicograph product of two graphs

The Lexicograph product of two simple graphs H_1 and H_2 with vertex sets $W_1 : \{u_1, u_2, ...\}$ *and* $W_2 : \{v_1, v_2, ...\}$ respectively, is defined as a graph with vertex set as one $W_1 \times W_2 : \{w_1, w_2, ...\}$ and two vertices $w_1 = (u_1, v_1)$ and $w_2 = (u_2, v_2)$ are adjacent if and only if either (i) $u_1 u_2 \in E(H_1)$ *or* (ii) $u_1 = u_2$ *and* $v_1 v_2 \in E(H_2)$.

The notation for this product graph is H_1 (l) H_2.

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2. Annihilator domination

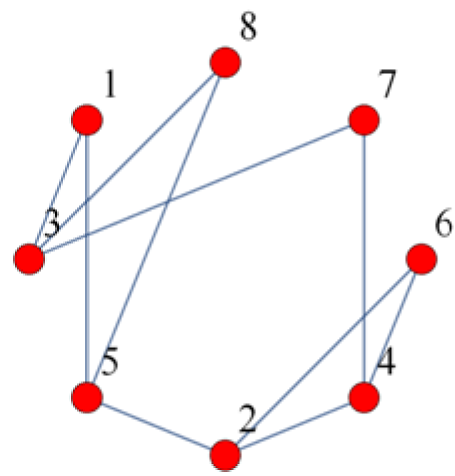
Definition: 2.1

If the induced sub graph <W- M> of a dominating set M of a graph H is a graph containing only isolated vertices, then the set is considered as an annihilator dominating set,.

The minimum cardinality of an annihilator domination set is its annihilator dominating number $\gamma_a(G)$ of G.

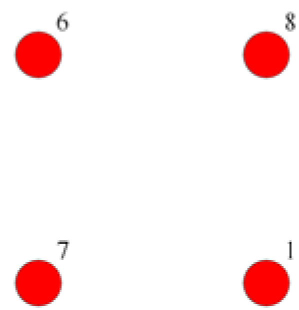
Illustration:

H:



Annihilator dominating Set $M = \{2, 3, 4, 5\}$

<W-M> :



The induced sub graph **<W-M>** of H is a graph with isolated vertices.

$\gamma_a(\mathrm{H}) = 4$

Fig. 1

We now gain numerous conclusions on the annihilator dominating set and its relation in terms of another domination characteristics.

By def. 2.1, the below is an instantaneous consequence.

Theorem 2. 2 : For a graph H, $\gamma(H) \leq \gamma_s(H) \leq \gamma_a(H)$

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Now, for several standard graphs, we estimate their annihilator domination number

Theorem 2.3 : $\gamma_a\left(K_{m,n}\right) = m$, if $K_{m,n}$ represents a complete bipartite graph, for 2≤ m≤n,

Theorem 2.4 : $\gamma_a\left(P_n\right) = \left[n / 2\right]$, if P_n denote a path on n-vertices, where [Y] presage the greatest integer ≤ Y.

Theorem 2. 5 : $\gamma_a\left(S_n\right) = 1$, if S_n indicates a star on n-vertices.

Theorem 2.6: $\gamma_a\left(W_n\right)=\begin{cases} n/2+1\,;\ If\ n\ is\ even \\ \dfrac{(n+1)}{2};\ if\ n\ is\ odd \end{cases}$, if W_n specify a wheel on n-vertices

Theorem 2.7 : $\gamma_a\left(C_n\right) = \lceil n/2 \rceil$, if C_n specify a cycle on n-vertices. where $\lceil x \rceil$ represents the smallest integer ≥ x.

Theorem 2.8 : $\gamma_a\left(T\right) \le n - p$, if T specify a tree on n-vertices for p ≥ 3 pendent vertices.

An amazing equation for a Graph H annihilator domination number in terms of split and domination numbers.

Theorem 2.9 : $\gamma_a\left(H\right) \ge \gamma_s\left(H\right) + \sum_{i-1}^{t}\gamma\left(H_i\right)$, where H_i's are the components of <W-M_s>, M_s being the split dominance set of Graph H of least cardinality.

3. ANNIHILATOR DOMINANCE OF PRODUCT GRAPHS

The annihilator dominating sets and expressions for the annihilator domination number of some product graphs that were previously established in section 1 are obtained in this work and come from the definitions 1.16, 1.17, and 1.18.

Finding a split domination set M_s of the product graph and then eliminating all remaining edges in the induced sub graph <W-M_s> is the first step in the process of obtaining the annihilator dominating set of a product graph and thus

$$\gamma_a(H_1\left(k\right)H_2] = \gamma_s[H_1\left(k\right)H_2] + \gamma[< W - M_s >]$$

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We now first prove the following result in order to get the annihilator domination number of a Kronecker product graph.

Theorem 3.1: $\gamma_a\left[\left(H_1(k)\ H_2\right)\right] \le min\left[\gamma_a(H_1)\ .\ |W_2|\ ,|W_1|.\ \gamma_a(H_2)\right]$, if H_1 and H_2 are two graphs without independent vertices,

Proof : Let H_1 and H_2 are two graphs with p_1 and p_2 vertices respectively.

Let $M_1 = \{\, u_{m1},\ u_{m2},...\ u_{mr}\}$ be an annihilator dominance set of least cardinality of H_1 and $M_2 = \{\, v_{m1},\ v_{m2},\ldots\ldots v_{ms}\}$ be an annihilator dominance set of least cardinality of H_2.

Now in the product graph $H_1(k)\ H_2$ consider the set of vertices,

$$M_a = \left\{\begin{matrix} (u_{m1},\ v_1),\ (u_{m1},\ v_2),\ldots\ldots\ (u_{m1},\ v_{p2}), \\ (u_{m2},\ v_1),\ (u_{m2},\ v_2),\ldots\ldots(u_{m2},\ v_{p2}), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (u_{mr},\ v_1),\ (u_{mr},\ v_2),\ldots.\ (u_{mr},\ v_{p2}) \end{matrix}\right\}$$

Since M_1 being an annihilator dominating set of H_1, the deletion of M_1 vertices from H_1 will makes the graph $\langle W_1 - M_1 \rangle$ be a set of isolated vertices.

Let M_a be a dominance set of $H_1(k)H_2$.

For, if (u,v) be any vertex in $\langle W\text{-}M_a \rangle$ of $H_1(k)H_2$, then u be adjacent with at least one vertex in $M_1 = \{\, u_{m1},\ u_{m2},...\ u_{mr}\}$, as this is the dominating set of H_1 being its annihilator dominance set.

If v is adjacent with some vertex v_k in H_2 ,surely there is at least one vertex v_k in H_2, where H_2 has no independent vertices), then (u, v) be adjacent to (u_{m_i}, v_k) in M_a.

Thus M_a is a dominance set.

Further to prove M_a be an annihilator dominating set, it can be established as mentioned below:

As we aware that M_1 is an annihilator dominating set of H_1 and the sub graph $\langle W_1 - M_1 \rangle$ is a graph with independent vertices.

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Consider, u_i, u_j be any two vertices in $<W_1–M_1>$. They are isolated vertices of $<W_1 - M_1>$.

If (u_1, v_1) and (u_2, v_2) are any two vertices in $<W-M_a>$, then $u_1, u_2 \neq u_{m_i}$, for any i.

As all the vertices of H_1 (k) H_2 whose first coordinates are vertices in M_1 are removed by virtue of the removal of M_a.

It follows that u_1, u_2 are not adjacent and consequently (u_1,v_1) and (u_2, v_2) are not adjacent in $<W – M_a>$.

Thus M_a is an annihilator dominating set of H_1 (k) H_2.

By the same argument it can be proved that the set of vertices

$$M_a' = \left\{ \begin{array}{l} (u_1, v_{m1}), (u_2, v_{m1}) \ldots\ldots\ldots (u_{p1}, v_{m1}), \\ (u_1, v_{m2}) (u_2, v_{m2}) \ldots\ldots\ldots\ldots (u_{p1}, v_{m2}), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (u_1, v_{ms}), (u_2, v_{ms}) \ldots\ldots (u_{p1}, v_{ms}) \end{array} \right\}$$ also be an annihilator dominance set of

$H_1(k)\, H_2$.

Thus it follows that

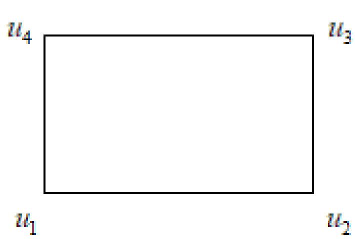
$\gamma_a [H_1(k)H_2] \leq \min [|M_a|, |M_a'|]$

$= \min [\gamma_a(H_1). |W_2|, |W_1|. \gamma_a(H_2)]$

Hence $\gamma_a [H_1(k)H_2] \leq \min [\gamma_a(H_1). |W_2|, |W_1|. \gamma_a(H_2)]$

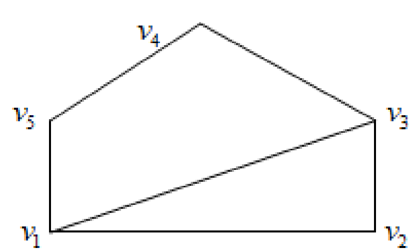
Illustration:

H_1:



Annihilator dominating Set
$H_1 = \{u_1, u_3\}$ and $\gamma_a(H_1) = 2$

H_2:



Annihilator dominating Set
$H_2 = \{v_1, v_3, v_5\}$ and $\gamma_a(H_2) = 3$

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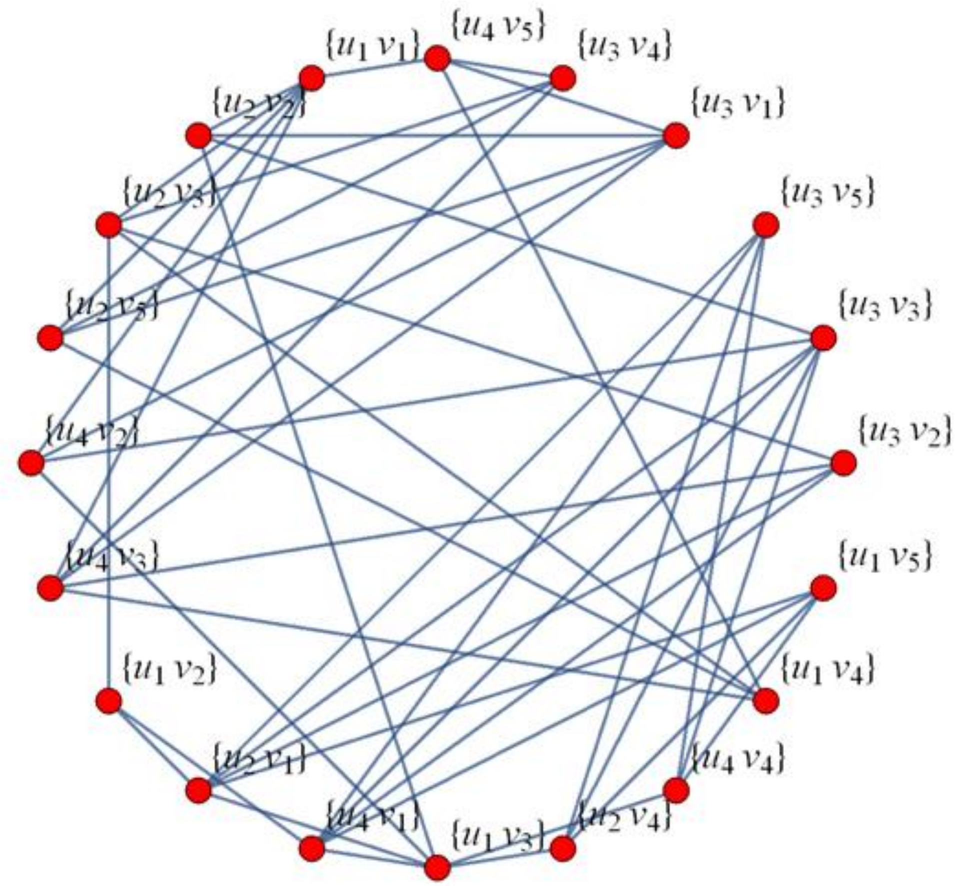
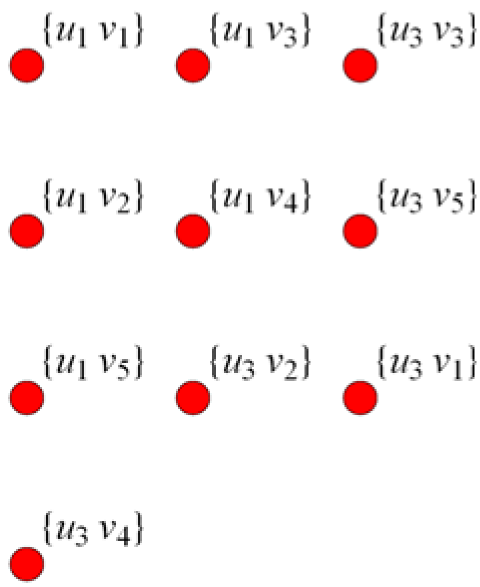


$H_1(K)\,H_2$



$< W - M_{a} >$:

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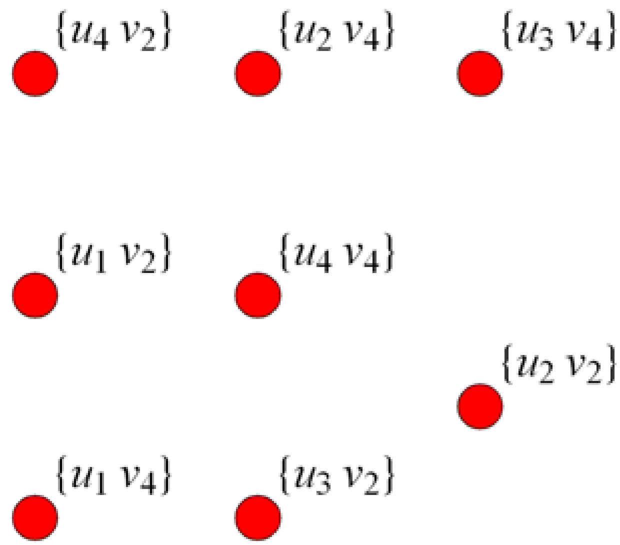
$$|M_a| = \gamma_a(H_1).\ |W_{2.5}| = 10.$$

$$M'_a = \{u_1, u_2, u_3, u_4\} \times \{v_1, v_3, v_5\}$$

$$= \begin{Bmatrix} (u_1, v_1), (u_1, v_3), (u_1, v_5), (u_2, v_1), (u_2, v_3), (u_2, v_5), \\ (u_3, v_1), (u_3, v_3), (u_3, v_5), (u_4, v_1), (u_4, v_3), (u_4, v_5) \end{Bmatrix}$$



$< W - M'_a >$

M'_a is an Annihilator dominating set

$$|M'_a| = |W_1|.\ \gamma_a(H_2) = 4.3 = 12$$

$$\gamma_a[H_1(K)H_2] \le \min\left[|M_a|, |M'_a|\right]$$

$$= \min[10, 12] \le 10$$

$$\gamma_a[H_1(K)H_2] \le 10$$

Fig. 2

We now obtain an upper bound for an annihilator dominating set of $H_1(L)H_2$

Theorem 3.2: If H_1 and H_2 are any two graphs without independent vertices,

Then $\gamma_a[H_1(L)H_2] \le \gamma_a(H_1)\ .\ |W_2| + |W_1|\ .\ \gamma_a(H_2) - \gamma_a(H_1)\ .\gamma_a(H_2)$

Proof: Let H_1 and H_2 are two graphs with p_1 and p_2 vertices.

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$$W(H_1) = \{u_1, u_2, \ldots\ldots\ldots\ldots u_{p1}\} = W_1 \; and$$
$$W(H_2) = \{v_1, v_2 \ldots\ldots\ldots\ldots v_{p2}\} = W_2 \; say$$

Let $M_1 = \{ u_{m1}, u_{m2} \ldots\ldots u_{mr}\}$ be the annihilator dominating set of H_1 and $M_2 = \{v_{m1}, v_{m2} \ldots\ldots\ldots v_{ms}\}$ be the annihilator dominating set of H_2

Now to obtain an annihilator dominating set of H_1 (L) H_2 we proceed as follows :

Consider the set of vertices

$$M = \begin{Bmatrix} (u_{m1}, v_1), (u_{m1}, v_2) \ldots\ldots\ldots (u_{m1}, v_{p2}), \\ (u_{m2}, v_1), (u_{m2}, v_2) \ldots\ldots\ldots (u_{m2}, v_{p2}), \\ \ldots\ldots\ldots \ldots\ldots\ldots \ldots\ldots\ldots \\ (u_{mr}, v_1), (u_{mr}, v_2) \ldots\ldots\ldots (u_{mr}, v_{p2}) \end{Bmatrix}$$

M is a dominating set of H_1 (L) H_2.

For, if (u,v) is a vertex of $H_1(L)H_2$, then u is adjacent with some vertex in $M_1 = \{ u_{m_1}, u_{m_2}, \ldots u_{m_r}\}$, since M_1 is the annihilator dominating set of H_1.

For this purpose, let u be adjacent with u_{m_i} , for some values of i.

Now v in H_2 is having adjacency with a vertex v_j in H_2, as H_2 is a simple graph without independent vertices

Thus the vertex (u, v) in $H_1(L)H_2$ is adjacent with (u_{m_i}, v_j).

However, the removal of this dominating set from H_1 (L) H_2 will give us the set of vertices W – M , where W is vertex set of H_1 (L) H_2.

The induced sub graph <W–M> is not an independent set of vertices. For, the removal of set of vertices of M enables only the elimination of the adjacency in the product graph, obtained by the use of the first part of the definition (2.14) viz., $w_1, w_2 \in H_1$ (L) H_2 are adjacent if $u_1 u_2 \in E(H_1)$.

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To eliminate the adjacency of the product graph in the remaining graph, we note that the adjacency will be due to the second part of the definition (2.14) viz., $w_1, w_2 \in H_1 (L) H_2$ are adjacent if $u_1 = u_2$ & $v_1 v_2 \in E(H_1)$.

For this purpose, we consider the set of vertices

$$M' = \left\{ \begin{array}{l} (u_1,\ v_{m1}),\ (u_2,\ v_{m1})\ldots\ldots (u_{p1}, v_{m1}), \\ \left(u_1, v_{m2}\right),\ \left(u_2, v_{m2}\right)\ldots\ldots \left(u_{p1}, v_{m2}\right), \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \left(u_1, v_{ms}\right),\ \left(u_2,\ v_{ms}\right)\ldots\ldots \left(u_{p1},\ v_{ms}\right) \end{array} \right\}$$

Proceeding in the same process as above

we can prove that M' is also a dominating set of $H_1(L)H_2$

Thus if $M_a = M \cup M'$, then since both M and M' are dominating sets of $H_1 (L) H_2$, M_a is also a dominating set of $H_1(L)H_2$.

More over M_a is an annihilator dominating set, because it eliminates the adjacency an account of 2.14 (a) as well as 2.14 (b) between any two vertices in the induced sub graph $< W-\{M \cup M^1\}>$

Thus $\gamma_a[H_1(L)H_2] \leq |M_a|$

$= | M \cup M^1 |$

But $| M \cup M^1| = | M | + | M^1| - | M \cap M^1|$

$= | M | + | M^1| - | M_1 \times M_2|$

where $| M | = \gamma_a(H_1) . p_2 = \gamma_a (H_1) . |W_2|$

$| M^1| = p_1 . \gamma_a(H_2) = |W_1| . \gamma_a(H_2)$

$| M_1 \times M_2| = rs = \gamma_a(H_1) . \gamma_a(H_2)$

Hence $\gamma_a [H_1(L)H_2] \leq \gamma_a (H_1) . |W_2| + |W_1| . \gamma_a(H_2) - \gamma_a (H_1). \gamma_a (H_2)$

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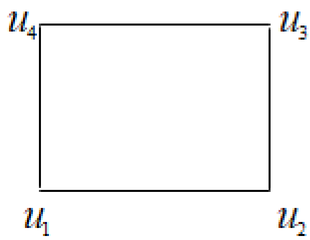
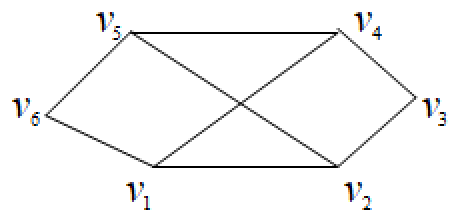
Illustration:

H_1 :



Annihilator dominating Set of $H_1 = \{u_1, u_3, u_5\} = M_1$ and $\gamma_a(H_1) = 3 = |M_1|$

H_2 :



Annihilator dominating Set $H_2 = \{v_1, v_3\} = M_2$ and $\gamma_a(H_2) = 2 = |M_2|$

$$
\begin{aligned}
M &= \{u_1, u_3\} \times \{v_1, v_2, v_3, v_4, v_5, v_6\} \\
&= \{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_1, v_6), \\
&\quad (u_3, v_1), (u_3, v_2), (u_3, v_3), (u_3, v_4), (u_3, v_5), (u_3, v_6)\}
\end{aligned}
$$

$$|M| = \gamma_a(H_1).|W_2| = 2.6 = 12$$

$$
\begin{aligned}
M' &= \{u_1, u_2, u_3, u_4\} \times \{v_1, v_3, v_5\} \\
&= \{(u_1, v_1), (u_1, v_3), (u_1, v_5), (u_2, v_1), (u_2, v_3), (u_2, v_5), \\
&\quad (u_3, v_1), (u_3, v_3), (u_3, v_5), (u_4, v_1), (u_4, v_3), (u_4, v_5)\}
\end{aligned}
$$

$$
\begin{aligned}
M_a = M \cup M' = \{&(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_1, v_5), (u_1, v_6), (u_2, v_1), \\
&(u_2, v_3), (u_2, v_5), (u_3, v_1), (u_3, v_2), (u_3, v_3), (u_3, v_4), (u_3, v_5), \\
&(u_3, v_6), (u_4, v_1), (u_4, v_3), (u_4, v_5)\}
\end{aligned}
$$

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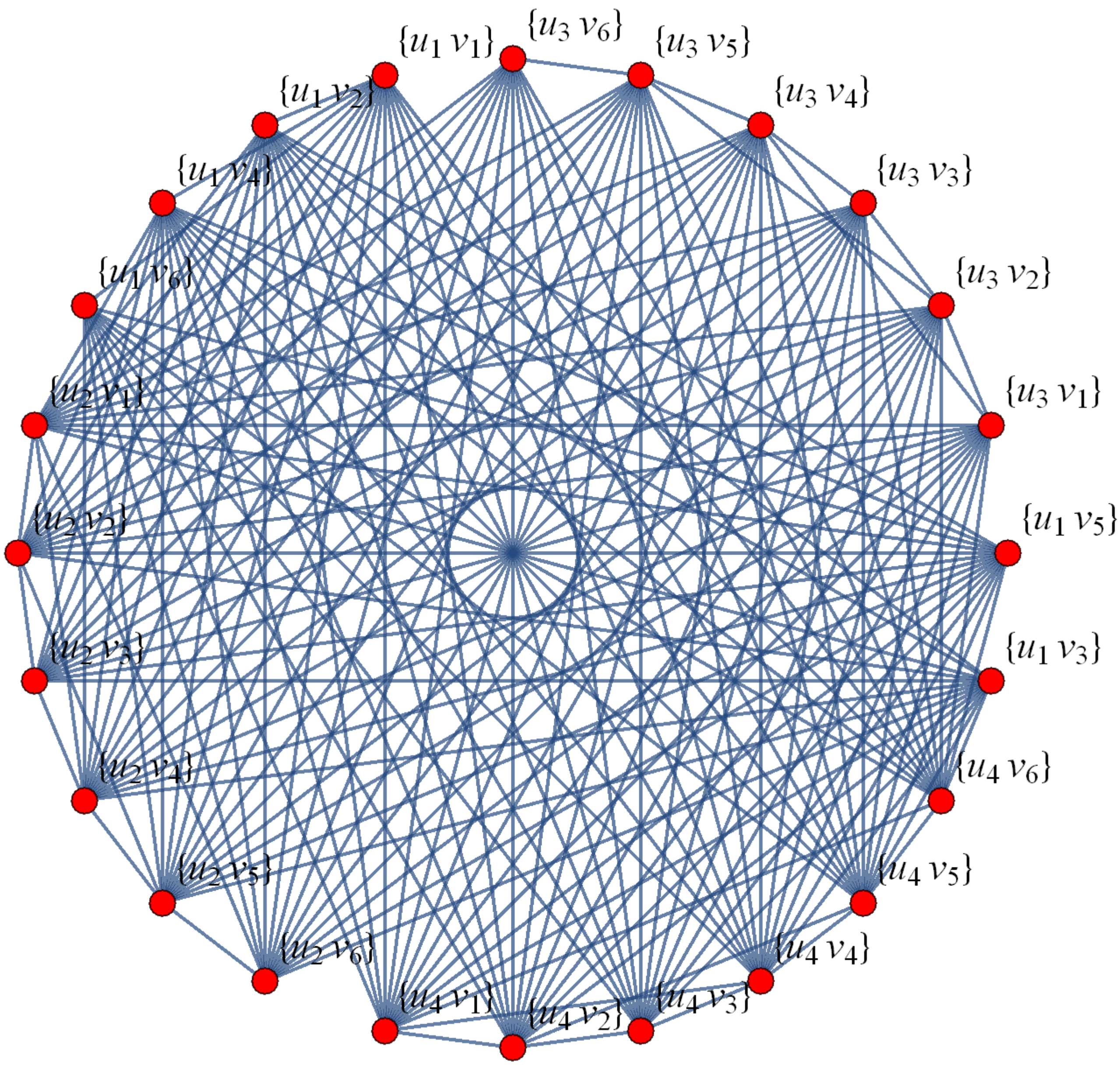

$H_1(L)H_2$

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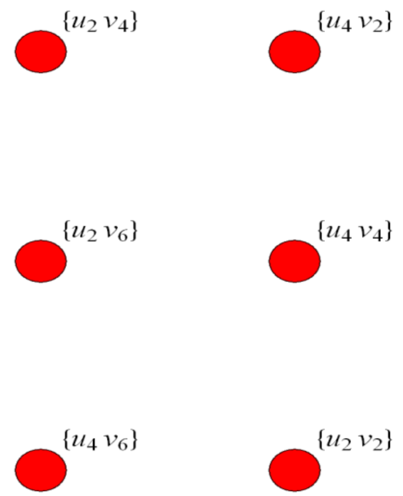


$< W - M \cup M' >$

D_a is an Annihilator dominating set

$$\gamma_a[H_1(L)H_2] \leq |M_a| = |M \cup M'|$$
$$= |M| + |M'| - |M \cap M'|$$
$$= |M| + |M'| - |M \times M'|$$
$$= 12 + 12 - 6 = 18$$
$$\gamma_a[H_1(L)H_2] \leq 18$$

Fig. 3

We now obtain an upperbound for the Lexicograph product graph.

We observe that from the definitions (1.6) & (1.7) that H_1 (C) H_2 is a sub graph of H_1(L) H_2. The following result is as an immediate extension of the previous result

Theorem 3.3: If H_1 and H_2 are two graphs without independent vertices, then

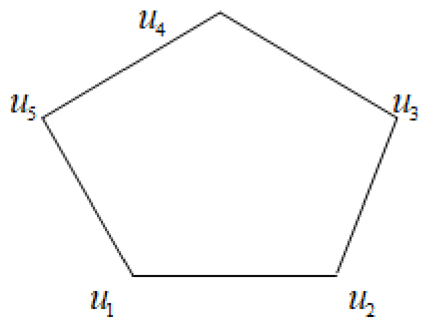
$$\gamma_a[H_1(C)H_2] \leq \gamma_a(H_1). |W_2| + |W_1|. \gamma_a(H_2) - \gamma_a(H_1). \gamma_a(H_2).$$

Proof : To get an annihilator dominance set of the graph $H_1(C)H_2$, we progress along the same lines as per the theorem (3.2) and it can be proved that the set M_a as defined in the theorem is an annihilator dominance set of H_1 (C) H_2.

Hence $\gamma_a[H_1(C)H_2] \leq \gamma_a(H_1). |W_2| + |W_1| . \gamma_a(H_2) - \gamma_a(H_1). \gamma_a(H_2)$

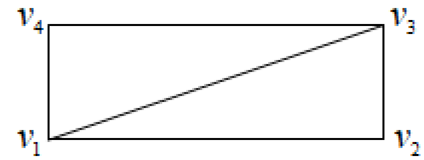
Illustration:

$\mathbf{H_1}$:

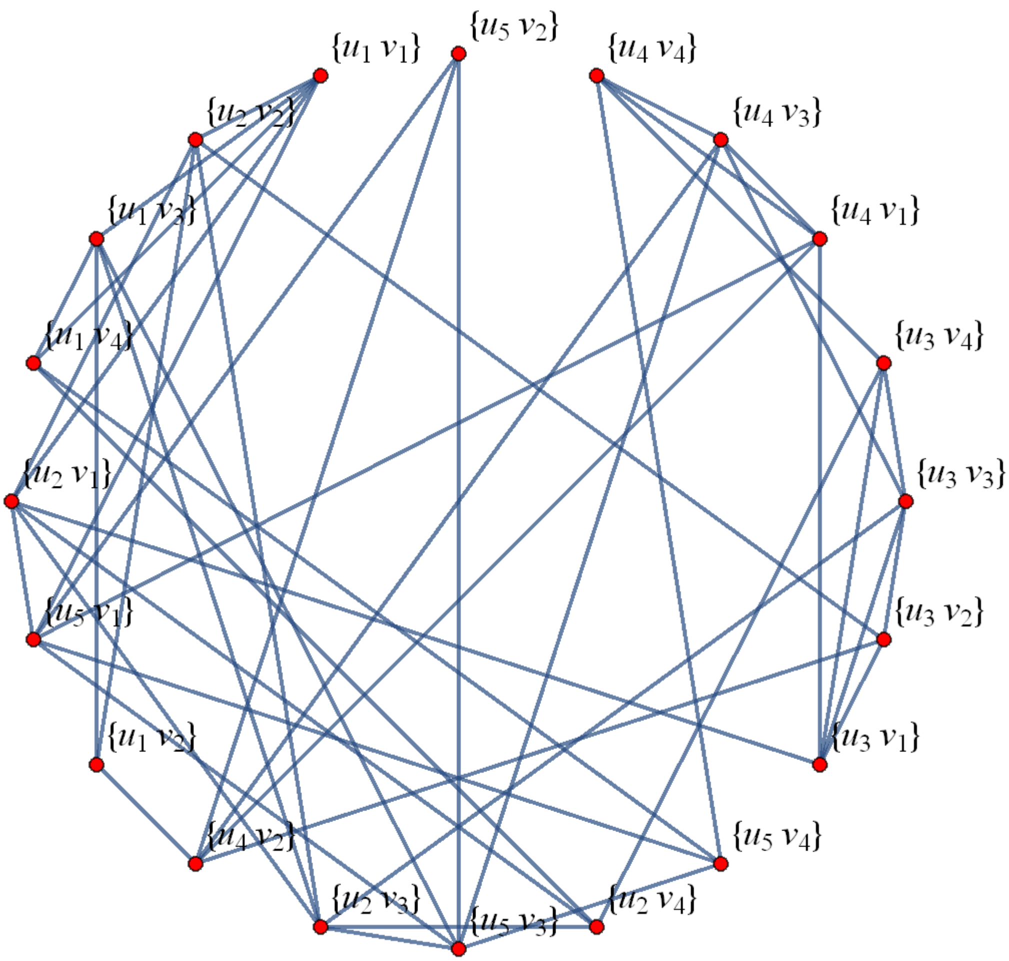


Annihilator dominating Set of
$H_1 = \{u_1, u_3, u_5\} = M_1$ and
$\gamma_a(\mathrm{H}_1) = 3 = |M_1|$

$\mathbf{H_2}$:



Annihilator dominating Set
$H_2 = \{v_1, v_3\} = M_2$ and
$\gamma_a(H_2) = 2 = |M_2|$



$\mathbf{H_1\ (C)\ H_2}$

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$$
\begin{aligned}
M &= \{u_1, u_3, u_5\} \times \{v_1, v_2, v_3, v_4\} \\
&= \{(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_3, v_1), (u_3, v_2), \\
&\quad (u_3, v_3), (u_3, v_4), (u_5, v_1), (u_5, v_2), (u_5, v_3), (u_5, v_4)\}
\end{aligned}
$$

$$\left|M\right| = \gamma_a(H_1).\left|W_2\right| = 3.4 = 12$$

$$
\begin{aligned}
M' &= \{u_1, u_2, u_3, u_4, u_5\} \times \{v_1, v_3\} \\
&= \{(u_1, v_1), (u_1, v_3), (u_2, v_1), (u_2, v_3), (u_3, v_1), \\
&\quad (u_3, v_3), (u_4, v_1), (u_4, v_3), (u_5, v_1), (u_5, v_3)\}
\end{aligned}
$$

$$\left|M\right| = \gamma_a(\mathrm{W}_1).\left|H_2\right| = 5.2 = 10$$

$$
\begin{aligned}
M_a = M \cup M' = \{&(u_1, v_1), (u_1, v_2), (u_1, v_3), (u_1, v_4), (u_2, v_1), (u_2, v_3), (u_3, v_1), \\
&(u_3, v_2), (u_3, v_3), (u_3, v_4), (u_4, v_1), (u_4, v_3), (u_5, v_1), (u_5, v_2), \\
&(u_3, v_6), (u_4, v_1), (u_4, v_3), (u_4, v_5)\}
\end{aligned}
$$

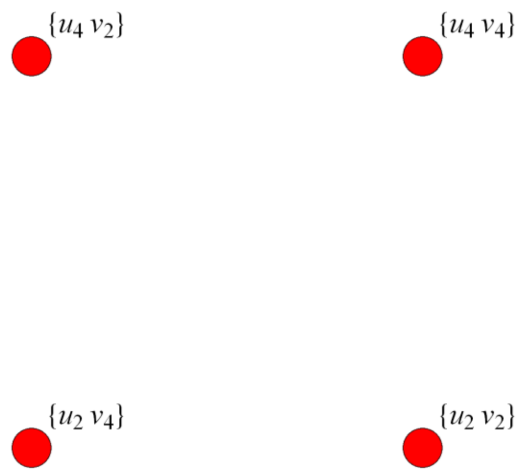

Fig. 4

4. Conclusion

There are various real world situations where the idea of annihilator dominating sets is useful. Three prominent applications that exhibit its usefulness are presented below:

1. **Pest Management in Agriculture:**

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Controlling insect populations is essential in agriculture to avoid extensive crop damage. Various pests interact and enhance each other's effects, making control measures tough. We can employ annihilator dominant sets to isolate particular pests by modeling pest interactions as a graph, where vertices represent distinct insect types and edges indicate interactions between them. Findings an annihilator dominance set in the graph aids in locating the pests whose eradication will cause the pest network to become disrupted, allowing for more focused pest control efforts.

2. **Managing Viral and Bacterial Infections:**

According to epidemiology, specific strains of bacteria and viruses combine to spread illness. Targeting certain strains is essential to fighting these illnesses because once they are destroyed; the remaining bacteria will become isolated and ultimately eradicated. By depicting every virus strain as a vertex and its interactions as edges in a graph, we mat utilize annihilator dominant sets to determine which strains require attention. Eliminating these strains from the graph will lead to isolated vertices, which will effectively stop the illness from spreading.

3. **Strategic Operations in Defense**

In military strategy, the communication between a unit's multiple camps or stations can affect how successful its operational strength is. It might be important to target particular camps in order to disrupt the enemy's operations, as doing so would cut off communication lines and limit the enemy's capabilities. Annihilator dominant sets can be used to determine which camps should be eliminated in order to isolate others by representing the network of camps as a graph, with vertices representing the camps and edges representing the communication channels. This tactic aids in improving defense efforts and achieving operational interruption.

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