PERIODICO di MINERALOGIA Volume 91, No. 3, 2022
ISSN: 0369-8963

Topological Operators Over Bipolar Intuitionistic Fuzzy Ideal and Bipolar Intuitionistic Anti Fuzzy Ideal of a Bp-Algebra

S. SIVAKAMINATHAN[1*], DR. K. GUNASEKARAN[2], AND DR. S. NANDAKUMAR[3]

[1,2]Ramanujan Research Centre, PG and Research Department of Mathematics
Government Arts College (Autonomous)
(Affiliated to Bharathidasan University, Tiruchirappalli), Kumbakonam – 612 002, Tamilnadu, India
[3]PG and Research Department of Mathematics, Government Arts College
(Affiliated to Bharathidasan University, Tiruchirappalli), Ariyalur- 621 713, Tamilnadu, India
**Corresponding Author*

Abstract
The concept of a bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are a new algebraic structure of BP-algebra and to use interior operator. The purpose of this study is to implement the fuzzy set theory and ideal theory of a BP-algebra. The relation between the operation of topological operators on bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are established.

Key words: *BP-algebra, fuzzy ideal, bipolar fuzzy ideal, bipolar intuitionistic fuzzy ideal, bipolar intuitionistic anti fuzzy ideal, interior operator.*

2010 Mathematics Subject Classification 08A72 Fuzzy algebraic structures

Introduction

The concept of fuzzy sets was initiated by I.A.Zadeh [14] then it has become a vigorous area of research in engineering, medical science, graph theory. S.S.Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was introduced by K.J.Lee [6] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree (0, 1] indicates that elements somewhat satisfies the property and the negative membership degree [-1, 0) indicates that elements somewhat satisfies the implicit counter property. The author W.R.Zhang [15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1998. K.Chakrabarty, R.Nanda and S.Biswas [3] investigated note on union and intersection of intuitionistic fuzzy sets. A.Rajeshkumar [13] was analyzed fuzzy groups and level subgroups. M.Palanivelrajan, K.Gunasekaran and S.Nandakumar [12] introduced the level operators on intuitionistic fuzzy primary ideal and semiprimary ideal. K.Gunasekaran, S.Nandakumar and S.Sivakaminathan [16] introduced the definition of bipolar intuitionistic fuzzy ideal of a BP-algebra.

Preliminaries

Definition: 2.1

Let A and B be any two bipolar intuitionistic fuzzy set $A = (\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N)$ and $B = (\mu_B^P, \mu_B^N, \nu_B^P, \nu_B^N)$ in X, we define

(i) $A \cap B = \{(x, \min(\mu_A^P(x), \mu_B^P(x)), \max(\mu_A^N(x), \mu_B^N(x)),$

ISSN: 0369-8963

$\max(\nu_A^P(x), \nu_B^P(x)), \min(\nu_A^N(x), \nu_B^N(x)))$ /x ∈ X},

(ii) A ∪ B ={(x, $\max(\mu_A^P(x), \mu_B^P(x)), \min(\mu_A^N(x), \mu_B^N(x)),$

$\min(\nu_A^P(x), \nu_B^P(x)), \max(\nu_A^N(x), \nu_B^N(x)))$ /x ∈ X},

(iii) $\overline{A}$= {(x,$\nu_A^P(x), \nu_A^N(x), \mu_A^P(x), \mu_A^N(x)$) / x ∈ X}.

Definition: 2.2

A bipolar intuitionistic fuzzy set A = {$\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$ / x ∈ X } of BP-algebra X is called a bipolar intuitionistic fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu_A^P(0) \geq \mu_A^P(x)$ and $\mu_A^N(0) \leq \mu_A^N(x)$,

(ii) $\mu_A^P(x) \geq \min\{\mu_A^P(x * y), \mu_A^P(y)\}$,

(iii) $\mu_A^N(x) \leq \max\{\mu_A^N(x * y), \mu_A^N(y)\}$,

(iv) $\nu_A^P(0) \leq \nu_A^P(x)$ and $\nu_A^N(0) \geq \nu_A^N(x)$,

(v) $\nu_A^P(x) \leq \max\{\nu_A^P(x * y), \nu_A^P(y)\}$,

(vi) $\nu_A^N(x) \geq \min\{\nu_A^N(x * y), \nu_A^N(y)\}$, for all x, y ∈ X.

Definition: 2.3

A bipolar intuitionistic fuzzy set A = {$\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$ / x ∈ X } of BP-algebra X is called a bipolar intuitionistic anti fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu_A^P(0) \leq \mu_A^P(x)$ and $\mu_A^N(0) \geq \mu_A^N(x)$,

(ii) $\mu_A^P(x) \leq \max\{\mu_A^P(x * y), \mu_A^P(y)\}$,

(iii) $\mu_A^N(x) \geq \min\{\mu_A^N(x * y), \mu_A^N(y)\}$,

(iv) $\nu_A^P(0) \geq \nu_A^P(x)$ and $\nu_A^N(0) \leq \nu_A^N(x)$,

(v) $\nu_A^P(x) \geq \min\{\nu_A^P(x * y), \nu_A^P(y)\}$,

(vi) $\nu_A^N(x) \leq \max\{\nu_A^N(x * y), \nu_A^N(y)\}$, for all x, y ∈ X.

Definition: 2.4

Let A is a bipolar intuitionistic fuzzy set of X, then the interior operator I is defined by I (A) = {(x, min $\mu_A^P(y)$, max $\mu_A^N(y)$, max $\nu_A^P(y)$, min $\nu_A^N(y)$) / x ∈ X , y ∈ X }.

Definition: 2.5

Let A is a bipolar intuitionistic fuzzy set of X, then the necessity operator □ is defined by □A = {(x, $\mu_A^P(x), \mu_A^N(x), 1 - \mu_A^P(x), -1 - \mu_A^N(x)$) / x ∈ X }.

Definition: 2.6

Let A is a bipolar intuitionistic fuzzy set of X, then the possibility operator ◊ is defined by ◊A = { (x, $1 - \nu_A^P(x), -1 - \nu_A^N(x), \nu_A^P(x), \nu_A^N(x)$) / x ∈ X) }.

Topological Operators on Bipolar Intuitionistic Fuzzy Ideal

Theorem: 3.1

If A is a bipolar intuitionistic fuzzy ideal of X, then I (A) is a bipolar intuitionistic fuzzy ideal of X.

https://doi.org/10.37896/pd91.3/91324

ISSN: 0369-8963

Proof: Given A is a bipolar intuitionistic fuzzy ideal of X.

Consider 0, x, y ∈ A.

(i) Now $\mu^P_{I(A)}(0) = \min\ \mu^P_A(x)$

$= \mu^P_A(a)$

$\geq \min\ \mu^P_A(a)$

$= \mu^P_{I(A)}(x)$

Therefore $\mu^P_{I(A)}(0) \geq \mu^P_{I(A)}(x)$.

Now $\mu^N_{I(A)}(0) = \max\ \mu^N_A(x)$

$= \mu^N_A(a)$

$\leq \max\ \mu^N_A(a)$

$= \mu^N_{I(A)}(x)$

Therefore $\mu^N_{I(A)}(0) \leq \mu^N_{I(A)}(x)$.

(ii) Now $\mu^P_{I(A)}(x) = \min\ \mu^P_A(a)$

$\geq \min \{ \min \{ \mu^P_A(a * b),\ \mu^P_A(b)\}\}$

$= \min \{ \min \mu^P_A(a * b),\ \min \mu^P_A(b)\}$

$= \min \{ \mu^P_{I(A)}(x * y),\ \mu^P_{I(A)}(y)\}$

Therefore $\mu^P_{I(A)}(x) \geq \min \{ \mu^P_{I(A)}(x * y),\ \mu^P_{I(A)}(y)\}$.

(iii) Now $\mu^N_{I(A)}(x) = \max\ \mu^N_A(a)$

$\leq \max \{ \max \{ \mu^N_A(a * b),\ \mu^N_A(b)\}\}$

$= \max \{ \max\ \mu^N_A(a * b),\ \max \mu^N_A(b)\}$

$= \max \{ \mu^N_{I(A)}(x * y),\ \mu^N_{I(A)}(y)\}$

Therefore $\mu^N_{I(A)}(x) \leq \max \{ \mu^N_{I(A)}(x * y),\ \mu^N_{I(A)}(y)\}$.

(iv) Now $\nu^P_{I(A)}(0) = \max \nu^P_A(x)$

$= \nu^P_A(a)$

$\leq \max \nu^P_A(a)$

$= \nu^P_{I(A)}(x)$

Therefore $\nu^P_{I(A)}(0) \leq \nu^P_{I(A)}(x)$.

Now $\nu^N_{I(A)}(0) = \min \nu^N_A(x)$

$= \nu^N_A(a)$

$\geq \min \nu^N_A(a)$

$= \nu^N_{I(A)}(x)$

Therefore $\nu^N_{I(A)}(0) \geq \nu^N_{I(A)}(x)$.

(v) Now $\nu^P_{I(A)}(x) = \max \nu^P_A(a)$

$\leq \max \{ \max \{ \nu^P_A(a * b),\ \nu^P_A(b)\}\}$

$= \max \{ \max \nu^P_A(a * b),\ \max \nu^P_A(b)\}$

$= \max \{ \nu^P_{I(A)}(x * y),\ \nu^P_{I(A)}(y)\}$

Therefore $\nu^P_{I(A)}(x) \leq \max \{ \nu^P_{I(A)}(x * y),\ \nu^P_{I(A)}(y)\}$.

(vi) Now $\nu^N_{I(A)}(x) = \min \nu^N_A(a)$

$\geq \min \{ \min \{ \nu^N_A(a * b),\ \nu^N_A(b) \}\}$

$= \min \{ \min \nu^N_A(a * b),\ \min \nu^N_A(b)\}$

$= \min \{ \nu^N_{I(A)}(x * y),\ \nu^N_{I(A)}(y)\}$

 https://doi.org/10.37896/pd91.3/91324

ISSN: 0369-8963

Therefore $\nu^N_{I(A)}(x) \geq \min\{ \nu^N_{I(A)}(x * y),\ \nu^N_{I(A)}(y)\}$.

Therefore I (A) is a bipolar intuitionistic fuzzy ideal of X.

Theorem: 3.2

If A is a bipolar intuitionistic fuzzy ideal of X, then I(I(A)) = I (A) is a bipolar intuitionistic fuzzy ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy ideal of X.

Consider 0, x, y ∈ A.

(i) Now $\mu^P_{I(I(A))}(0) = \min\ \mu^P_{I(A)}(x)$

$= \min(\min \mu^P_A(0))$

$\geq \min\ \mu^P_A(0)$

$= \mu^P_{I(A)}(x)$

Therefore $\mu^P_{I(I(A))}(0) \geq \mu^P_{I(A)}(x)$.

Now $\mu^N_{I(I(A))}(0) = \max\ \mu^N_{I(A)}(x)$

$= \max(\max \mu^N_A(0))$

$\leq \max\ \mu^N_A(0)$

$= \mu^N_{I(A)}(x)$

Therefore $\mu^N_{I(I(A))}(0) \leq \mu^N_{I(A)}(x)$.

(ii) Now $\mu^P_{I(I(A))}(x) = \min\ \mu^P_{I(A)}(a)$

$= \min(\min \mu^P_A(x))$

$= \min(\mu^P_A(a))$

$\geq \min\{\min\{\mu^P_A(a * b),\ \mu^P_A(b)\}\}$

$= \min\{\min \mu^P_A(a * b),\ \min \mu^P_A(b)\}$

$= \min\{\mu^P_{I(A)}(x * y),\ \mu^P_{I(A)}(y)\}$

Therefore $\mu^P_{I(I(A))}(x) \geq \min\{\mu^P_{I(A)}(x * y),\ \mu^P_{I(A)}(y)\}$.

(iii) Now $\mu^N_{I(I(A))}(x) = \max\ \mu^N_{I(A)}(a)$

$= \max(\max \mu^N_A(x))$

$= \max(\mu^N_A(a))$

$\leq \max\{\max\{\mu^N_A(a * b),\ \mu^N_A(b)\}\}$

$= \max\{\max\ \mu^N_A(a * b),\ \max \mu^N_A(b)\}$

$= \max\{\mu^N_{I(A)}(x * y),\ \mu^N_{I(A)}(y)\}$

Therefore $\mu^N_{I(I(A))}(x) \leq \max\{\mu^N_{I(A)}(x * y),\ \mu^N_{I(A)}(y)\}$.

(iv) Now $\nu^P_{I(I(A))}(0) = \max \nu^P_{I(A)}(x)$

$= \max(\max \nu^P_A(0))$

$\leq \max \nu^P_A(0)$

$= \nu^P_{I(A)}(x)$

Therefore $\nu^P_{I(I(A))}(0) \leq \nu^P_{I(A)}(x)$.

Now $\nu^N_{I(I(A))}(0) = \min \nu^N_{I(A)}(x)$

$= \min(\min \nu^N_A(0))$

$\geq \min \nu^N_A(0)$

$= \nu^N_{I(A)}(x)$

 https://doi.org/10.37896/pd91.3/91324

Therefore $\nu^N_{I(I(A))}(0) \geq \nu^N_{I(A)}(x)$.

(v) Now $\nu^P_{I(I(A))}(x)$ = max $\nu^P_{I(A)}(a)$

= max (max $\nu^P_A(x)$)

= max ($\nu^P_A(a)$)

$\leq$ max { max { $\nu^P_A(a*b),\ \nu^P_A(b)$}}

= max { max $\nu^P_A(a*b)$, max $\nu^P_A(b)$}

= max { $\nu^P_{I(A)}(x*y),\ \nu^P_{I(A)}(y)$}

Therefore $\nu^P_{I(I(A))}(x) \leq$ max { $\nu^P_{I(A)}(x*y),\ \nu^P_{I(A)}(y)$}.

(vi) Now $\nu^N_{I(I(A))}(x)$ = min $\nu^N_{I(A)}(a)$

= min (min $\nu^N_A(x)$)

= min ($\nu^N_A(a)$)

$\geq$ min { min { $\nu^N_A(a*b),\ \nu^N_A(b)$}}

= min { min $\nu^N_A(a*b)$, min $\nu^N_A(b)$}

= min { $\nu^N_{I(A)}(x*y),\ \nu^N_{I(A)}(y)$}

Therefore $\nu^N_{I(I(A))}(x) \geq$ min { $\nu^N_{I(A)}(x*y),\ \nu^N_{I(A)}(y)$}.

Therefore I(I(A)) = I (A) is a bipolar intuitionistic fuzzy ideal of X.

Theorem: 3.3

If A and B are bipolar intuitionistic fuzzy ideals of X, then I (A ∩ B) = I (A) ∩ I(B) is a bipolar intuitionistic fuzzy ideal of X.

Proof: Given A and B are bipolar intuitionistic fuzzy ideals of X.

Consider 0, x, y ∈ A ∩ B then 0, x, y ∈ A and 0, x, y ∈ B.

(i) Now $\mu^P_{I(A\cap B)}(0)$ = min $\mu^P_{A\cap B}(x)$

= min (min ($\mu^P_A(x),\ \mu^P_B(x)$))

= min (min $\mu^P_A(x)$, min $\mu^P_B(x)$)

= min ($\mu^P_A(a),\ \mu^P_B(a)$)

$\geq$ min (min $\mu^P_A(a)$, min $\mu^P_B(a)$)

= min ($\mu^P_{I(A)}(x),\ \mu^P_{I(B)}(x)$)

= $\mu^P_{I(A)\cap I(B)}(x)$

Therefore $\mu^P_{I(A\cap B)}(0) \geq \mu^P_{I(A)\cap I(B)}(x)$.

Now $\mu^N_{I(A\cap B)}(0)$ = max $\mu^N_{A\cap B}(x)$

= max (max ($\mu^N_A(x), \mu^N_B(x)$))

= max (max $\mu^N_A(x)$, max $\mu^N_B(x)$)

= max ($\mu^N_A(a), \mu^N_B(a)$)

$\leq$ max (max $\mu^N_A(a)$, max $\mu^N_B(a)$)

= max ($\mu^N_{I(A)}(x),\ \mu^N_{I(B)}(x)$)

= $\mu^N_{I(A)\cap I(B)}(x)$

Therefore $\mu^N_{I(A\cap B)}(0) \leq \mu^N_{I(A)\cap I(B)}(x)$.

(ii) Now $\mu^P_{I(A\cap B)}(x)$ = min $\mu^P_{A\cap B}(a)$

= min (min ($\mu^P_A(a),\ \mu^P_B(a)$))

= min (min $\mu^P_A(a)$, min $\mu^P_B(a)$)

 https://doi.org/10.37896/pd91.3/91324

$\geq \min(\min\{\min\{\mu_A^P(a*b),\ \mu_A^P(b)\}\},$
$\min\{\min\{\mu_B^P(a*b),\ \mu_B^P(b)\}\})$
$= \min\{\min(\min\{\mu_A^P(a*b),\ \mu_A^P(b)\}),\ \min(\min\{\mu_B^P(a*b),\ \mu_B^P(b)\})\}$
$= \min\{\min(\min\{\mu_A^P(a*b),\ \mu_B^P(a*b)\}),\ \min(\min\{\mu_A^P(b),\ \mu_B^P(b)\})\}$
$= \min\{\min(\min \mu_A^P(a*b), \min \mu_B^P(a*b)),\ \min(\min \mu_A^P(b),\ \min \mu_B^P(b))\}$
$= \min\{\min(\mu_{I(A)}^P(x*y),\ \mu_{I(B)}^P(x*y)),\ \min(\mu_{I(A)}^P(y),\ \mu_{I(B)}^P(y))\}$
$= \min\{\mu_{I(A)\cap I(B)}^P(x*y),\ \mu_{I(A)\cap I(B)}^P(y)\}$

Therefore $\mu_{I(A\cap B)}^P(x) \geq \min\{\mu_{I(A)\cap I(B)}^P(x*y),\ \mu_{I(A)\cap I(B)}^P(y)\}$.

(iii) Now $\mu_{I(A\cap B)}^N(x) = \max\ \mu_{A\cap B}^N(a)$
$= \max(\max(\mu_A^N(a),\ \mu_B^N(a)))$
$= \max(\max \mu_A^N(a),\ \max \mu_B^N(a))$
$\leq \max(\max\{\max\{\mu_A^N(a*b),\ \mu_A^N(b)\}\},$
$\max\{\max\{\mu_B^N(a*b),\ \mu_B^N(b)\}\})$
$= \max\{\max(\max\{\mu_A^N(a*b),\ \mu_A^N(b)\}),\ \max(\max\{\mu_B^N(a*b),\ \mu_B^N(b)\})\}$
$= \max\{\max(\max\{\mu_A^N(a*b),\ \mu_B^N(a*b)\}),\ \max(\max\{\mu_A^N(b),\ \mu_B^N(b)\})\}$
$= \max\{\max(\max \mu_A^N(a*b),\ \max \mu_B^N(a*b)),\ \max(\max \mu_A^N(b), \max \mu_B^N(b))\}$
$= \max\{\max(\mu_{I(A)}^N(x*y),\ \mu_{I(B)}^N(x*y)),\ \max(\mu_{I(A)}^N(y),\ \mu_{I(B)}^N(y))\}$
$= \max\{\mu_{I(A)\cap I(B)}^N(x*y),\ \mu_{I(A)\cap I(B)}^N(y)\}$

Therefore $\mu_{I(A\cap B)}^N(x) \leq \max\{\mu_{I(A)\cap I(B)}^N(x*y),\ \mu_{I(A)\cap I(B)}^N(y)\}$.

(iv) Now $\nu_{I(A\cap B)}^P(0) = \max\ \nu_{A\cap B}^P(x)$
$= \max(\max(\nu_A^P(x),\ \nu_B^P(x)))$
$= \max(\max \nu_A^P(x),\ \max \nu_B^P(x))$
$= \max(\nu_A^P(a),\ \nu_B^P(a))$
$\leq \max(\max \nu_A^P(a),\ \max \nu_B^P(a))$
$= \max(\nu_{I(A)}^P(x),\ \nu_{I(B)}^P(x))$
$= \nu_{I(A)\cap I(B)}^P(x)$

Therefore $\nu_{I(A\cap B)}^P(0) \leq \nu_{I(A)\cap I(B)}^P(x)$.

Now $\nu_{I(A\cap B)}^N(0) = \min \nu_{A\cap B}^N(x)$
$= \min(\min(\nu_A^N(x),\ \nu_B^N(x)))$
$= \min(\min \nu_A^N(x),\ \min \nu_B^N(x))$
$= \min(\nu_A^N(a),\ \nu_B^N(a))$
$\geq \min(\min \nu_A^N(a),\ \min \nu_B^N(a))$
$= \min(\nu_{I(A)}^N(x),\ \nu_{I(B)}^N(x))$
$= \nu_{I(A)\cap I(B)}^N(x)$

Therefore $\nu_{I(A\cap B)}^N(0) \geq \nu_{I(A)\cap I(B)}^N(x)$.

(v) Now $\nu_{I(A\cap B)}^P(x) = \max\ \nu_{A\cap B}^P(a)$
$= \max(\max(\nu_A^P(a),\ \nu_B^P(a)))$
$= \max(\max \nu_A^P(a),\ \max \nu_B^P(a))$
$\leq \max(\max\{\max\{\nu_A^P(a*b),\ \nu_A^P(b)\}\},$
$\max\{\max\{\nu_B^P(a*b),\ \nu_B^P(b)\}\})$
$= \max\{\max(\max\{\nu_A^P(a*b),\ \nu_A^P(b)\}),\ \max(\max\{\nu_B^P(a*b),\ \nu_B^P(b)\})\}$
$= \max\{\max(\max\{\nu_A^P(a*b), \nu_B^P(a*b)\}),\ \max(\max\{\nu_A^P(b),\ \nu_B^P(b)\})\}$

 https://doi.org/10.37896/pd91.3/91324

$= \max \{ \max (\max \nu_A^P(a*b),\ \max \nu_B^P(a*b)),\ \max (\max \nu_A^P(b),\ \max \nu_B^P(b))\}$
$= \max \{ \max (\nu_{I(A)}^P(x*y),\ \nu_{I(B)}^P(x*y)),\ \max (\nu_{I(A)}^P(y),\ \nu_{I(B)}^P(y))\}$
$= \max \{ \nu_{I(A)\cap I(B)}^P(x*y),\ \ \nu_{I(A)\cap I(B)}^P(y)\}$

Therefore $\nu_{I(A\cap B)}^P(x) \le \max \{ \nu_{I(A)\cap I(B)}^P(x*y),\ \ \nu_{I(A)\cap I(B)}^P(y)\}$.

(vi) Now $\nu_{I(A\cap B)}^N(x) = \min \nu_{A\cap B}^N(a)$
$= \min (\min (\nu_A^N(a),\ \nu_B^N(a)))$
$= \min (\min \nu_A^N(a),\ \ \min \nu_B^N(a))$
$\ge\ \min (\min \{ \min \{ \nu_A^N(a*b),\ \nu_A^N(b) \}\},$
$\min \{ \min \{ \nu_B^N(a*b),\ \nu_B^N(b) \}\})$
$= \min \{ \min (\min \{ \nu_A^N(a*b),\ \nu_A^N(b)\}), \min (\min \{ \nu_B^N(a*b),\ \nu_B^N(b) \}) \}$
$= \min \{ \min (\min \{ \nu_A^N(a*b),\ \nu_B^N(a*b)\}),\ \min (\min \{ \nu_A^N(b)\ ,\ \nu_B^N(b) \}) \}$
$= \min \{ \min (\min \nu_A^N(a*b),\ \min \nu_B^N(a*b)),\ \min (\min \nu_A^N(b),\ \min \nu_B^N(b)) \}$
$= \min \{ \min (\nu_{I(A)}^N(x*y),\ \nu_{I(B)}^N(x*y)),\ \min (\nu_{I(A)}^N(y),\ \nu_{I(B)}^N(y)) \}$
$= \min \{ \nu_{I(A)\cap I(B)}^N(x*y),\ \ \nu_{I(A)\cap I(B)}^N(y)\}$

Therefore $\nu_{I(A\cap B)}^N(x) \ge \min \{ \nu_{I(A)\cap I(B)}^N(x*y),\ \ \nu_{I(A)\cap I(B)}^N(y)\}$.

Therefore I (A ∩ B) = I (A) ∩ I(B) is a bipolar intuitionistic fuzzy ideal of X.

Theorem: 3.4

If A is a bipolar intuitionistic fuzzy ideal of X, then □(I(A)) = I(□(A)) is a bipolar intuitionistic fuzzy ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy ideal of X.
Consider 0, x, y ∈ A.

(i) Now $\mu_{\square(I(A))}^P(0) = \mu_{I(A)}^P(0)$
$= \min\ \mu_A^P(x)$
$=\ \mu_A^P(a)$
$\ge \min \mu_A^P(a)$
$=\ \min\ \mu_{\square(A)}^P(a)$
$=\ \mu_{I(\square(A))}^P(x)$

Therefore $\mu_{\square(I(A))}^P(0) \ge\ \mu_{I(\square(A))}^P(x)$.

Now $\mu_{\square(I(A))}^N(0) = \mu_{I(A)}^N(0)$
$= \max\ \mu_A^N(x)$
$=\ \mu_A^N(a)$
$\le\ \max \mu_A^N(a)$
$= \max\ \mu_{\square(A)}^N(a)$
$=\ \mu_{I(\square(A))}^N(x)$

Therefore $\mu_{\square(I(A))}^N(0) \le\ \mu_{I(\square(A))}^N(x)$.

(ii) Now $\mu_{\square(I(A))}^P(x) = \mu_{I(A)}^P(x)$
$= \min\ \mu_A^P(a)$
$\ge \min \{ \min \{ \mu_A^P(a*b),\ \mu_A^P(b)\}\}$
$= \min \{ \min \{ \mu_{\square(A)}^P(a*b),\ \mu_{\square(A)}^P(b)\}\}$
$= \min \{ \min\ \mu_{\square(A)}^P(a*b), \min\ \mu_{\square(A)}^P(b)\}$

https://doi.org/10.37896/pd91.3/91324

$= \min \{ \mu^{P}_{I(\square(A))}(x * y),\ \ \mu^{P}_{I(\square(A))}(y)\}$

Therefore $\mu^{P}_{\square(I(A))}(x) \geq \min \{ \mu^{P}_{I(\square(A))}(x * y),\ \mu^{P}_{I(\square(A))}(y)\}$.

(iii) Now $\mu^{N}_{\square(I(A))}(x) = \mu^{N}_{I(A)}(x)$

$= \max\ \mu^{N}_{A}(a)$

$\leq \max \{ \max \{ \mu^{N}_{A}(a * b),\ \mu^{N}_{A}(b)\}\}$

$= \max \{ \max \{ \mu^{N}_{\square(A)}(a * b),\ \mu^{N}_{\square(A)}(b)\}\}$

$= \max \{ \max \mu^{N}_{\square(A)}(a * b),\ \max \mu^{N}_{\square(A)}(b)\}$

$= \max \{ \mu^{N}_{I(\square(A))}(x * y),\ \ \mu^{N}_{I(\square(A))}(y)\}$

Therefore $\mu^{N}_{\square(I(A))}(x) \leq \max \{ \mu^{N}_{I(\square(A))}(x * y),\ \ \mu^{N}_{I(\square(A))}(y)\}$.

(iv) Now $\nu^{P}_{\square(I(A))}(0) = 1 - \mu^{P}_{I(A)}(0)$

$= \max (1 - \mu^{P}_{A}(x))$

$= 1 - \mu^{P}_{A}(a)$

$\leq \max (1 - \mu^{P}_{A}(a))$

$= \max (1 - \mu^{P}_{\square(A)}(a))$

$= \max \nu^{P}_{\square(A)}(a)$

$= \nu^{P}_{I(\square(A))}(x)$

Therefore $\nu^{P}_{\square(I(A))}(0) \leq \nu^{P}_{I(\square(A))}(x)$.

Now $\nu^{N}_{\square(I(A))}(0) = 1 - \mu^{N}_{I(A)}(0)$

$= \min (1 - \mu^{N}_{A}(x))$

$= 1 - \mu^{N}_{A}(a)$

$\geq \min (1 - \mu^{N}_{A}(a))$

$= \min (1 - \mu^{N}_{\square(A)}(a))$

$= \min \nu^{N}_{\square(A)}(a)$

$= \nu^{N}_{I(\square(A))}(x)$

Therefore $\nu^{N}_{\square(I(A))}(0) \geq \nu^{N}_{I(\square(A))}(x)$.

(v) Now $\nu^{P}_{\square(I(A))}(x) = 1 - \mu^{P}_{I(A)}(x)$

$= \max (1 - \mu^{P}_{A}(a))$

$\leq \max \{ \max \{ 1 - \mu^{P}_{A}(a * b),\ 1 - \ \mu^{P}_{A}(b)\}\}$

$= \max \{ \max \{ 1 - \mu^{P}_{\square(A)}(a * b),\ 1 - \ \mu^{P}_{\square(A)}(b)\}\}$

$= \max \{ \max (1 - \mu^{P}_{\square(A)}(a * b)), \max (1 - \ \mu^{P}_{\square(A)}(b))\}$

$= \max \{ \max \nu^{P}_{\square(A)}(a * b),\ \max \nu^{P}_{\square(A)}(b)\}$

$= \max \{ \nu^{P}_{I(\square(A))}(x * y),\ \ \nu^{P}_{I(\square(A))}(y)\}$

Therefore $\nu^{P}_{\square(I(A))}(x) \leq \max \{ \nu^{P}_{I(\square(A))}(x * y),\ \ \nu^{P}_{I(\square(A))}(y)\}$.

(vi) Now $\nu^{N}_{\square(I(A))}(x) = 1 - \mu^{N}_{I(A)}(x)$

$= \min (1 - \mu^{N}_{A}(a))$

$\geq \min \{ \min \{ 1 - \mu^{N}_{A}(a * b),\ 1 - \ \mu^{N}_{A}(b)\}\}$

$= \min \{ \min \{ 1 - \mu^{N}_{\square(A)}(a * b),\ 1 - \ \mu^{N}_{\square(A)}(b)\}\}$

$= \min \{ \min (1 - \mu^{N}_{\square(A)}(a * b)),\ \min (1 - \ \mu^{N}_{\square(A)}(b))\}$

$= \min \{ \min \nu^{N}_{\square(A)}(a * b),\ \ \min \nu^{N}_{\square(A)}(b)\}$

$= \min \{ \nu^{N}_{I(\square(A))}(x * y),\ \nu^{N}_{I(\square(A))}(y)\}$

 https://doi.org/10.37896/pd91.3/91324

ISSN: 0369-8963

Therefore $\nu^{N}_{\square(\mathrm{I(A)})}(x) \geq \min \{ \nu^{N}_{\mathrm{I}(\square(\mathrm{A}))}(x * y),\ \nu^{N}_{\mathrm{I}(\square(\mathrm{A}))}(y)\}$.

Therefore □(I(A)) = I(□(A)) is a bipolar intuitionistic fuzzy ideal of X.

Theorem: 3.5

If A is a bipolar intuitionistic fuzzy ideal of X, then ◊(I(A)) = I(◊(A)) is a bipolar intuitionistic fuzzy ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy ideal of X.

Consider 0, x, y ∈ A.

(i) Now $\mu^{P}_{\lozenge(\mathrm{I(A)})}(0) = 1 - \nu^{P}_{\mathrm{I(A)}}(0)$

$= \min (1 - \nu^{P}_{\mathrm{A}}(x))$

$= 1 - \nu^{P}_{\mathrm{A}}(a)$

$\geq \min (1 - \nu^{P}_{\mathrm{A}}(a))$

$= \min (1 - \nu^{P}_{\lozenge(\mathrm{A})}(a))$

$= \min \mu^{P}_{\lozenge(\mathrm{A})}(a)$

$= \mu^{P}_{\mathrm{I}(\lozenge(\mathrm{A}))}(x)$

Therefore $\mu^{P}_{\lozenge(\mathrm{I(A)})}(0) \geq \mu^{P}_{\mathrm{I}(\lozenge(\mathrm{A}))}(x)$.

Now $\mu^{N}_{\lozenge(\mathrm{I(A)})}(0) = 1 - \nu^{N}_{\mathrm{I(A)}}(0)$

$= \max (1 - \nu^{N}_{\mathrm{A}}(0))$

$= 1 - \nu^{N}_{\mathrm{A}}(a)$

$\leq \max (1 - \nu^{N}_{\mathrm{A}}(a))$

$= \max (1 - \nu^{N}_{\lozenge(\mathrm{A})}(a))$

$= \max \mu^{N}_{\lozenge(\mathrm{A})}(a)$

$= \mu^{N}_{\mathrm{I}(\lozenge(\mathrm{A}))}(x)$

Therefore $\mu^{N}_{\lozenge(\mathrm{I(A)})}(0) \leq \mu^{N}_{\mathrm{I}(\lozenge(\mathrm{A}))}(x)$.

(ii) Now $\mu^{P}_{\lozenge(\mathrm{I(A)})}(x) = 1 - \nu^{P}_{\mathrm{I(A)}}(x)$

$= \min (1 - \nu^{P}_{\mathrm{A}}(a))$

$\geq \min \{ \min \{ 1 - \nu^{P}_{A}(a * b),\ 1 - \nu^{P}_{A}(b)\}\}$

$= \min \{ \min \{ 1 - \nu^{P}_{\lozenge(\mathrm{A})}(a * b),\ 1 - \nu^{P}_{\lozenge(\mathrm{A})}(b)\}\}$

$= \min \{ \min (1 - \nu^{P}_{\lozenge(\mathrm{A})}(a * b)),\ \min (1 - \nu^{P}_{\lozenge(\mathrm{A})}(b))\}$

$= \min \{ \min \mu^{P}_{\lozenge(\mathrm{A})}(a * b),\ \min \mu^{P}_{\lozenge(\mathrm{A})}(b)\}$

$= \min \{ \mu^{P}_{\mathrm{I}(\lozenge(\mathrm{A}))}(x * y),\ \mu^{P}_{\mathrm{I}(\lozenge(\mathrm{A}))}(y)\}$

Therefore $\mu^{P}_{\lozenge(\mathrm{I(A)})}(x) \geq \min \{ \mu^{P}_{\mathrm{I}(\lozenge(\mathrm{A}))}(x * y),\ \mu^{P}_{\mathrm{I}(\lozenge(\mathrm{A}))}(y)\}$.

(iii) Now $\mu^{N}_{\lozenge(\mathrm{I(A)})}(x) = 1 - \nu^{N}_{\mathrm{I(A)}}(x)$

$= \max (1 - \nu^{N}_{\mathrm{A}}(a))$

$\leq \max \{ \max \{ 1 - \nu^{N}_{A}(a * b),\ 1 - \nu^{N}_{A}(b) \}\}$

$= \max \{ \max \{ 1 - \nu^{N}_{\lozenge(\mathrm{A})}(a * b),\ 1 - \nu^{N}_{\lozenge(\mathrm{A})}(b) \}\}$

$= \max \{ \max (1 - \nu^{N}_{\lozenge(\mathrm{A})}(a * b)),\ \max (1 - \nu^{N}_{\lozenge(\mathrm{A})}(b))\}$

$= \max \{ \max \mu^{N}_{\lozenge(\mathrm{A})}(a * b),\ \max \mu^{N}_{\lozenge(\mathrm{A})}(b)\}$

 https://doi.org/10.37896/pd91.3/91324

$= \max \{ \mu^N_{I(\lozenge(A))}(x * y) , \ \mu^N_{I(\lozenge(A))}(y)\}$

Therefore $\mu^N_{\lozenge(I(A))}(x) \leq \ \max \{ \mu^N_{I(\lozenge(A))}(x * y) , \ \mu^N_{I(\lozenge(A))}(y)\}$.

(iv) Now $\nu^P_{\lozenge(I(A))}(0) = \ \nu^P_{I(A)}(0)$

$= \max \nu^P_A(x)$

$= \ \nu^P_A(a)$

$\leq \max \nu^P_A(a)$

$= \max \nu^P_{\lozenge(A)}(a)$

$= \nu^P_{I(\lozenge(A))}(x)$

Therefore $\nu^P_{\lozenge(I(A))}(0) \leq \nu^P_{I(\lozenge(A))}(x)$.

Now $\nu^N_{\lozenge(I(A))}(0) = \nu^N_{I(A)}(0)$

$= \min \nu^N_A(x)$

$= \ \nu^N_A(a)$

$\geq \ \min \nu^N_A(a)$

$= \min \nu^N_{\lozenge(A)}(a)$

$= \nu^N_{I(\lozenge(A))}(x)$

Therefore $\nu^N_{\lozenge(I(A))}(0) \geq \nu^N_{I(\lozenge(A))}(x)$.

(v) Now $\nu^P_{\lozenge(I(A))}(x) = \ \nu^P_{I(A)}(x)$

$= \max \nu^P_A(a)$

$\leq \max \{ \max \{ \nu^P_A(a * b), \ \nu^P_A(b)\}\}$

$= \max \{ \max \{ \nu^P_{\lozenge(A)}(a * b), \ \nu^P_{\lozenge(A)}(b)\}\}$

$= \max \{ \max \nu^P_{\lozenge(A)}(a * b), \ \ \max \nu^P_{\lozenge(A)}(b)\}$

$= \max \{ \nu^P_{I(\lozenge(A))}(x * y), \ \ \nu^P_{I(\lozenge(A))}(y)\}$

Therefore $\nu^P_{\lozenge(I(A))}(x) \leq \max \{ \nu^P_{I(\lozenge(A))}(x * y), \ \ \nu^P_{I(\lozenge(A))}(y)\}$.

(vi) Now $\nu^N_{\lozenge(I(A))}(x) = \nu^N_{I(A)}(x)$

$= \min \nu^N_A(a)$

$\geq \min \{ \min \{ \nu^N_A(a * b), \ \nu^N_A(b) \}\}$

$= \min \{ \min \{ \nu^N_{\lozenge(A)}(a * b), \ \nu^N_{\lozenge(A)}(b) \}\}$

$= \min \{ \min \nu^N_{\lozenge(A)}(a * b), \min \nu^N_{\lozenge(A)}(b)\}$

$= \min \{ \nu^N_{I(\lozenge(A))}(x * y), \ \nu^N_{I(\lozenge(A))}(y)\}$

Therefore $\nu^N_{\lozenge(I(A))}(x) \geq \min \{ \nu^N_{I(\lozenge(A))}(x * y), \ \nu^N_{I(\lozenge(A))}(y)\}$.

Therefore ◊(I(A)) = I(◊(A)) is a bipolar intuitionistic fuzzy ideal of X.

Topological Operators on Bipolar Intuitionistic anti Fuzzy Ideal

 https://doi.org/10.37896/pd91.3/91324

Theorem: 4.1

If A is a bipolar intuitionistic anti fuzzy ideal of X, then I (A) is a bipolar intuitionistic anti fuzzy ideal of X.

Theorem: 4.2

If A is a bipolar intuitionistic anti fuzzy ideal of X, then I(I(A)) = I (A) is a bipolar intuitionistic anti fuzzy ideal of X.

Theorem: 4.3

If A and B are bipolar intuitionistic anti fuzzy ideals of X, then I (A ∩ B) = I (A) ∩ I(B) is a bipolar intuitionistic anti fuzzy ideal of X.

Theorem: 4.4

If A is a bipolar intuitionistic anti fuzzy ideal of X, then □(I(A)) = I(□(A)) is a bipolar intuitionistic anti fuzzy ideal of X.

Theorem: 4.5

If A is a bipolar intuitionistic anti fuzzy ideal of X, then ◊(I(A)) = I(◊(A)) is a bipolar intuitionistic anti fuzzy ideal of X.

CONCLUSION

In this paper, the main idea of a bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are a new algebraic structure of BP-algebra and it is used through the interior operator. The aim of this study is implemented. The relation between the operation of topological operators on bipolar intuitionistic fuzzy ideal and bipolar intuitionistic anti fuzzy ideal are discussed. We believe that our ideas can also be applied for other algebraic system.

REFERENCES

[1]. S.Abdullah and M.M.M. Aslam, Bipolar fuzzy ideals in LA-semigroups, World Appl. Sci. J., 17.12 (2012), 1769-1782.

[2]. S.S.Ahn and J.S.Han, On BP-algebra, Hacettepe Journal of Mathematics and Statistics, 42 (2013), 551-557.

[3]. K.Chakrabarthy, Biswas R.Nanda, A note on union and intersection of intuitionistic fuzzy sets, Notes on intuitionistic fuzzy sets, 3(4), 1997.

[4]. K.J.Lee and Y.B.Jun, Bipolar fuzzy a-ideals of BCI-algebras, Commun. Korean Math. Soc., 26.4 (2011), 531-542.

[5]. K.J.Lee, Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull. Malays. Math. Sci. Soc., 32.3 (2009), 361-373.

[6]. K.J.Lee, Bipolar-valued fuzzy sets and their basic operations, Proc. Int. Conf., Bangkok, Thailand, 2007, 307-317.

[7]. J,Meng and X.Guo, On fuzzy ideals in BCK/BCI-algebras, Fuzzy Sets and Systems,

149.3 (2005), 509-525.

[8]. Osama Rashad EI-Gendy, Bipolar fuzzy α-ideal of BP –algebra, American Journal of Mathematics and Statistics 2020, 10(2): 33-37.

[9]. M.Palanivelrajan and S.Nandakumar, Intuitionistic fuzzy primary and semiprimary ideal, Indian Journal of Applied Research, Vol.1, 2012, No. 5, 159-160.

[10]. M.Palanivelrajan and S.Nandakumar, Some properties of intuitionistic fuzzy primary and semi primary ideals, Notes on intuitionistic fuzzy sets, 18, No.3 (2012), 68-74.

[11]. M.Palanivelrajan and S.Nandakumar, Some operations of intuitionistic fuzzy primary and semiprimary ideal, Asian journal of algebra 5, No.2 (2012), 44-49.

[12]. M.Palanivelrajan, K.Gunasekaran and S.Nandakumar, Level operators over intuitionistic fuzzy primary ideal and semiprimary ideal, Advances in Fuzzy sets and systems, Vol.15, No.2 (2013), 95-111.

[13]. A. Rajeshkumar, Fuzzy Algebra: Volume I (Publication division,University of Delhi).

[14]. L.A.Zadeh, Fuzzy sets, Information Control, 8 (1965), 338-353.

[15]. W.R.Zhang, Bipolar fuzzy sets, Part I, Proc. of FUZZ-IEEE, 2 (1998), 835-840.

[16]. K.Gunasekaran, S.Nandakumar and S.Sivakaminathan, Bipolar intuitionistic fuzzy α-ideal of a BP-algebra, Journal of Shanghai Jiaotong University, 17(8), 2021, 8-24.