On – Quasi Posia (N, K) Class and (N, K) – Quasi Class Q Composition Operators

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Abstract

In this article, various properties of quasi posi class A operators on Hilbert space,(n,k) – quasi class q composition operators on space are characterized.

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Introduction

Let H be a separable Complex Hilbert Space and C be the set of Complex numbers. Let B(H) indicate the C^* - algebra of all bounded linear operators act upon H. $T \in B(H)$ is stated to be P hyponormal for p>0 if $(T^*T)^p - (TT^*)^p \geq 0$ .When p = 1,T is termed as hyponormal .T is labeled paranormal if $\|Tx\|^2 \leq \|T^2x\|\|x\|$ for all x. T is termed as normaloid if $\|T^n\| = \|T\|^n$ for all n$\in$ N .Class A is defined by $\|T\| = (T^*T)^{1/2}$ which is considered to be the absolute value of T. Recently yuan and Gao introduced class A(n) which is $|T^{1+n}|^{2/1+n} \geq |T|^2$ operators and n – paranormal operators i.e,$\|T^{1+n}x\| \geq \|Tx\|$ for each and every unit vectors $x \in H$ and for some positive integer n. Let H,K be the Complex Hilbert spaces and $H \otimes K$ which is the tensor product of H,K i.e the Completion of the algebraic tensor product of H,K being the inner product

$$\langle x_1 \otimes y_1, x_2 \otimes y_2 \rangle = \langle x_1, x_2 \rangle \langle y_1, y_2 \rangle \; x_1, x_2 \in H, y_1, y_2 \in k .$$

Let $T \in B(H)$ and $S \in B(K), T \otimes S \in (H \otimes K)$ indicates the tensor product of T and S i.e $(T \otimes S)(x \otimes y) = Tx \otimes Sy for \; x \in H, y \in K.$

An operator T$\in B(H)$ is called as Hyponormal if $T^*T \geq TT^*$,P – Hyponormal if $(T^*T)^P \geq (TT^*)^P$ for a positive integer P. P – posinormal if $C^2(T^*T)^p \geq (TT^*)^p$ for some c >0 and 0<p<1,.n perinormal if $T^{*n}T^n \geq (T^*T)^n$ for all $n \geq 2$

An operator $T \in B(H)$ is claimed to be positive if $\langle Tx, x \rangle \geq 0$ for all $x \in H$.

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An operator is labeled as Paranormal if $\|Tx\|^2 \leq \|T^2x\|\|x\|$ for all $x \in H$.

An operator is professed to be (P,K) quasi hyponormal if $T^{*k}(T^*T)^P \geq (TT^*)^P T^k, 0 < P \leq 1$.

An operator is vowed to be (P,K) quasi posinormal if $T^{*k}C^2(T^*T)^p \geq (TT^*)^P T^k$ for some c>0, $0 < P \leq 1$ for some integer k>0.

An Operator $T \in B(H)$ is claimed to be quasi paranormal if $\|T^2x\|^2 \leq \|T^3x\|\|Tx\|$ for all $x \in H$; with $\|x\| = 1$.

Let n, k be positive integers. An operator $T \in B(H)$ is said to be

- Class A if $|T^2| \geq |T|^2$
- K – quasi class A if $T^{*k}|T^2|T^k \geq T^{*k}|T|^2 T^k$.

(n, k) – quasi paranormal if $\left\|T^{1+n}\left(T^k x\right)\right\|^{\frac{1}{1+n}} \left\|T^k x\right\|^{\frac{n}{1+n}} \geq \left\|T\left(T^k x\right)\right\|$ for all $x \in H$.

- Class A if $|T^2| \geq |T^*|^2$
- K – quasi * - class A if $T^{*k}|T^2|T^k \geq T^{*k}|T^*|^2 T^k$

K – quasi - * - paranormal if $\left\|T^2\left(T^k x\right)\right\|^{\frac{1}{2}} \left\|T^k x\right\|^{\frac{1}{2}} \geq \left\|T^*\left(T^k x\right)\right\|$ for all $x \in H$.

(n,k) quasi -*- paranormal if $\left\|T^{1+n}\left(T^k x\right)\right\|^{\frac{1}{1+n}} \left\|T^k x\right\|^{\frac{n}{1+n}} \geq \left\|T^* T^k x\right\|$ for all $x \in H$.

The close association which exists in the numerous classes of operators are stated by class $A \subseteq k - quasi - * - classA \subseteq k - quasi - * - paranormal$
Hyponormal $\Rightarrow$ class A $\Rightarrow$quasi – class A.
$\Rightarrow$quasi – paranormal $\Rightarrow$ k – quasi paranormal.
For n= 1
(n,K) – qusi paranormal – k – quasi paranormal and
(n,k) quasi - * - paranormal $\Rightarrow$paranormal$\Rightarrow$ k – quasi - * - paranormal.
(n,k) – quasi class Q$\Rightarrow$k – quasi class Q and
(n,k) quasi - *- class Q $\Rightarrow$k – quasi * class Q

Let $L^2(X, A, \mu)$ be a σ finite measure space. Let T be a measurable transformation on X. Then the Composition operator C on the space $L^2(\mu)$ engendered by T is given by Cf = foT for each $f \in L^2(\mu)$.

Let u be an approximately bounded function .Then the weighted Composition operator $W = W_{U,T}$ on the space $L^2(\mu)$ engendered by u and T is given by Wf = u. foT for each $f \in L^2(\mu)$.

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A transformation T is measurable if $T^{-1}(A) \in A$ for any $A \in$ A. The measurable transformation T is said to be nonsingular if $\mu(T^{-1}(A)) = 0 whenever \mu(A) = 0$ for $A \in$ A. If T is a measurable transformation then T^n is also a measurable transformation. If T is nonsingular, then we may say that μT^{-1} is entirely continuous with respect to μ and hence $\mu(T^{-1})^n$ becomes completely continuous with respect to μ. Henceforth Random – Nikodym theorem insists a unique non- negative which is necessarily bounded measurable function h_n such that $\mu(T^{-1})^n(A) = \int_A h_n d\mu$ for every $A \in$ A.

and h_n is labelled the nth order Random – Nikodym derivative and is indicated by $\dfrac{d\mu(T^{-1})^n}{d\mu}$.

It can be stated that $h_n = h.hoT^{-1}.hoT^{-2}......hoT^{-(n-1)}$ and $h_n = h_{n-1}.hoT^{-(n-1)}$ where μ is non negative.

Definition 1.1 Let $T \in B(H)$ be a (p,k) quasi – posi – n – perinormal operator on Complex Hilbert space H for all $n \geq 2$ such that $T^{*k}(C^2(T^{*n}T^n)^P - (T^*T)^{np})T^k \geq 0$
For a positive integer k,0<P<1 and fixed constant C>0.

T is termed as quasi class A if $T^*\left|T^2\right|T \geq T^*|T|^2 T$

Quasi posi class A if $(C^2\left|T^2\right| - |T|^2)T \geq 0$ .K quasi class A if $T^{*k}\left|T^2\right|T^k \geq T^{*k}|T|^2 T$. K quasi posi Class A if $T^{*k}(C^2\left|T^2\right| - |T|^2)T^k \geq 0$.

A bounded operator T is called quasi A(n,k) if $T^{*k}\left|T^n\right|T^k \geq T^{*k}|T|^n T^k$ i.e $T^{*k}(\left|T^n\right| - |T|^n)T^k \geq 0$ .T is said to be k quasi posi A(n,k) class A if $T^{*k}(C^2\left|T^n\right| - |T|^n)T^k \geq 0$. $T \in B(H)$ is professed as k – quasi posi class A(n) operators for positive integer n and k if $T^{*k}(C^2\left|T^{1+n}\right|^{\frac{2}{1+n}} - \left|T^2\right|)T^k \geq 0$

An operator T professed to be which is called (P,K) quasi posi class A if $T^{*k}(C^2\left|T^2\right|^p - |T|^{2p})T^k \geq 0$

An operator $T \in L(H)$ is also said to be quasi $*classAifT^*\left|T^2\right|T \geq T^*|T|^2 T$. An operator $T \in L(H)$ is declared to be k quasi - *- class A if $T^{*k}\left|T^2\right|T^k \geq T^{*k}|T|^2 T^k$

<u>Theorem 1.2</u>

Let $T \in L(H)$ be K – quasi posi – class A(n) operator for positive integer k and n. If $M \subset H$ is an invariant subspace of T, then the restriction T/M is also a K – quasi posi class A(n) Operator.

<u>Proof:</u>

Let P be the orthogonal projection of H onto M. Let $T_1 = T/M$.

Then $T^k P = PT^k P$ and $T_1 = PTP/M$. As T is a k – quasi – posi – class A(n) operator . We have $PT^{*k}(C^2|T^{1+n}|^{\frac{2}{1+n}} T^k P \geq PT^{*K}|T|^2 T^k P$. As

$$PT^{*k}(C^2|T^{1+n}|^{\frac{2}{1+n}} T^k P = PT^{*k}P(C^2 T^{*n+1}T^{n+1})^{\frac{1}{n+1}} PT^k P$$

$$= PT^{*K}(C^2 PT^{*n+1}T^{n+1}P)^{\frac{1}{n+1}} T^k P$$

$$= \begin{pmatrix} C^2 T_1^{*k}|T_1^{n+1}|^{\frac{2}{n+1}} T_1^k & 0 \\ 0 & 0 \end{pmatrix}$$

Through Hansen's inequality

$$PT^{*k}C^2|T|^2 T^k P = PT^{*k}PT^*C^2TPT^k P = \begin{pmatrix} T_1^{*k}C^2 T_1^2 T_1^k & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} T_1^{*k}C^2|T_1^{n+1}|^{\frac{2}{n+1}} T_1^k & 0 \\ 0 & 0 \end{pmatrix} \geq PT^{*k}C^2|T^{n+1}|^{\frac{2}{n+1}} T^k P \geq PT^{*k}C^2|T|^2 T^k P = \begin{pmatrix} T_1^{*k}C^2|T_1|^2 T_1^k & 0 \\ 0 & 0 \end{pmatrix}$$

i.e T_1 is also -a – k quasi posi class A(n) operator.

Tensor Products for k – quasi posi class A(n) operators

Let $T \otimes S$ indicate the tensor product on the product space $H \otimes K$ for nonzero $T \in B(H)$ and $S \in B(K)$ .The operation of taking tensor products T$\otimes S$ helps to preserve many properties of $T \in B(H)$ and $S \in B(K)$. $T \otimes S$ which is normal if and only if T &S are normal, however, there exists para normal operator $T \in B(H)$ and $S \in B(K)$ such as $T \otimes S$ is not always paranormal.

Duggal exclaimed that fornon zero $T \in B(H)$ and $S \in B(K)$,

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$T \otimes S$ is P – hyponormal if and only if T,S are P – hyponormal. This result was enlarged to P – quasi hyponormal operators. Class A operators,* - class A operators, log – hyponormal operators and class A (s,t) operators.

The following theorem brings out the necessary and ample condition for

$T \otimes S$ to be a k – quasi class A(n) 0perator. Both T & s are nonzero operators.

Theorem 2.1

Let $T \in B(H)$ and $S \in B(K)$ be non zero operators. Then

$T \otimes S \in B(H \otimes K)$ is a k – quasi – posi class A(n) operator if and only if one of the following assertions

is detained..

1. $T^{k+1} = 0 or S^{k+1} = 0$
2. T and S are k – quasi posi class A(n) operators.

Proof:

It is clear that $T \otimes S$ is k- quasi posiclass A(n) operators if and only if

$$(T \otimes S)^{*k}(C^2\left|(T \otimes S)^{1+n}\right|^{\frac{2}{1+n}} - \left|T \otimes S\right|^2)(T \otimes S)^K \geq 0$$

$$\Leftrightarrow T^{*k}(C^2\left|T^{1+n}\right|^{\frac{2}{1+n}} - \left|T\right|^2)T^k \otimes S^{*k}\left|S^{1+n}\right|^{\frac{2}{1+n}}S^k + T^{*k}C^2\left|T\right|^2 T^k \otimes S^{*k}(\left|S^{1+n}\right|^{\frac{2}{1+n}} - \left|S\right|^2)S^k \geq 0$$

$$\Leftrightarrow T^{*k}C^2\left|T^{1+n}\right|^{\frac{2}{1+n}}T^k \otimes S^{*k}(\left|S^{1+n}\right|^{\frac{2}{1+n}} - \left|S\right|^2)S^k + T^{*k}C^2(\left|T^{1+n}\right|^{\frac{2}{1+n}} - \left|T\right|^2)T^k \otimes S^{*k}\left|S\right|^2 S^k \geq 0$$

To prove the necessary .Assume that $T \otimes S$ is a k – quasi – class A(n) operator. Let $x \in H$ and $y \in K$ be arbitrary. Then

$$\left\langle T^{*k}C^2(\left|T^{1+n}\right|^{\frac{2}{1+n}} - \left|T\right|^2)T^k x, x\right\rangle\left\langle S^{*k}\left|S^{1+n}\right|^{\frac{2}{1+n}}S^k y, y\right\rangle + \left\langle T^{*k}C^2\left|T^2\right|T^k x, x\right\rangle\left\langle S^{*k}\left|S^{1+n}\right|^{\frac{2}{1+n}} - \left|S\right|^2)S^k y, y\right\rangle \geq 0$$

It suffices to prove that if (1) does not detain ,then (2) detains. Assume that $T^{k+1} \neq 0$ and $S^{k+1} \neq 0$. Assume that T is not a k – quasi posi class A(n) operator, then

there exists $x_0 \in H$ such as $\left\langle T^{*k}C^2(\left|T^{1+n}\right|^{\frac{2}{!+n}} - \left|T\right|^2)T^k x_0, x_0\right\rangle = \alpha < 0$ and

$$\left\langle T^{*k}C^2\left|T\right|^2 T^k x_0, x_0\right\rangle = \beta > 0$$

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$\alpha\left\langle S^{*k}\left|S^{1+n}\right|^{\frac{2}{1+n}}S^{k}y,y\right\rangle+\beta\left\langle S^{*k}(\left|S^{1+n}\right|^{\frac{2}{1+n}}-\left|S\right|^{2})S^{k}y,y\right\rangle\geq 0$ for all y $\in k$ However S is k quasi class A(n) operator. S = $\begin{pmatrix} S_1 & S_2 \\ 0 & S_3 \end{pmatrix}$ on $k=\overline{ran(S^k)}\oplus Ker(S^{*k})$ .Where S_1 is a class A(n) operators. Let P be the orthogonal k on to $\overline{ran(S^k)}$

$$\begin{pmatrix} \left|S_1^{1+n}\right|^{\frac{2}{1+n}} & 0 \\ 0 & 0 \end{pmatrix} = ((SP)^{*(1+n)}(SP)^{1+n})^{\frac{1}{1+n}} = (P\left|S^{1+n}\right|^{2}P)^{\frac{1}{1+n}} \geq P\left|S^{1+n}\right|^{\frac{2}{1+n}}P$$

$(\alpha+\beta)\left\langle S^{*k}\left|S_1^{1+n}\right|^{\frac{2}{1+n}}S^{k}y,y\right\rangle\geq\beta\left\langle S^{*k}\left|S\right|^{2}S^{k}y,y\right\rangle$ for all y $\in K$.

$(\alpha+\beta)\left\langle\left|S_1^{1+n}\right|^{\frac{2}{1+n}}\eta,\eta\right\rangle\geq\beta\left\langle\left|S_1\right|^{2}\eta,\eta\right\rangle$ for all $\eta\in\overline{ran(S^k)}$

Take supremum over all $\eta\in\overline{ran(S_1)}^{k}$

$(\alpha+\beta)\left\|S_1^{1+n}/^{\frac{1}{1+n}}\right\|^{2}\geq\beta\left\|S_1\right\|^{2}$ Self adjoint operatorsnormaloid.

$(\alpha+\beta)\left\|S_1\right\|^{2}\geq\beta\left\|S_1\right\|^{2}\Rightarrow S_1=0$ Since $S^{K+1}y=0$ for all y $\in H$. We have $S^{K+1}=0$ The assumption Contradicts $S^{K+1}=0$. Therefore T should be a k – quasi class A(n) operator.A similar argument proves that S as well as k – quasi class A(n) operator.

Proposition 2.2 Let X be a non – empty set and Let A be a σ algebra on X. Let μ and μT^{-1} can be measured on A and let h:X $\rightarrow[0,\infty]$ be a measurable function. In that case, the following are equivalents.

1. μT^{-1} is completely continuous with respect to μ and h is Random – Nikodym derivative of μT^{-1} completely in respect of μ .
2. For every measurable function $f:X\rightarrow[0,\infty]$, the equality $\int_X fd\mu T^{-1}=\int_X fhd\mu$ detains.

The Conditional expectation operator $E(./T^{-1}(A))=E(f)$ is defined for each non-negative function f in $L^p(1\leq p\prec\infty)$ and is distinctively determined by the following group of conditions.

1. E(f) is T^{-1}(**A)** measurable.
2. If B is any T^{-1}(A) measurable Set for which $\int_A fd\mu$

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Converges then we have $\int_A f d\mu = \int_A f d\mu = \int_A E(f) d\mu$ E is the projection operator on to the closure of the range of the composition operator C on $L^2(\mu)$.

Lemma 2.3 Let P be the projection of $L^2(X,$ A, $\mu)$ on to $\overline{R(C)}$ .Then

1. $C^*Cf = hfand \quad CC^* f = (hoT)Pf$ for all $f \in L^2(\mu)$.

2. $\overline{R(C)} = \{f \in L^2(\mu) : fisT^{-1}(A)measurable\}$.

3. If f is $T^{-1}($ **A)** measurable and g and fg are related to $L^2(\mu)$,

Then $P(\int g) = \int p(g), \int neednotbein\ L^2(\mu)$.

In this paper we have obtained the conditions for composition and Weighted Composition operators to be (n,k) – class Q and (n,k) – quasi _*- class Q operators with reference to the expectation operator and Random -= Nikodym derivative h.

Composition Operators

Let C be the composition operator and C^* be its adjoint which is specified by

$C^* f = h.E(f)oT^{-1}$

Proposition 2.4: For every $n \in$ N ,

i. $(C^*C)^n f = h^n f$

$ii.(CC^*)^n f = (hoT)^n P(f)$

iii) E is the identity operator on $L^2(\mu)$

iff $T^{-1}(A) = A$

Lemma 2.5 1 An operator $T \in B(H)$ is (n,k) – quasi class Q if and only if

$$T^{*k}T^{*(1+n)}T^{(1+n)}T^k - (1+n)T^{*k}T^*TT^k + nT^{*k}T^k \geq 0$$

Theorem 2.6 1 Let C be a composition operator on $L^2(\mu)$ .Then C would be (n,k) quasi class Q iff

$$h^{k+n+1} - (1+n)h^{k+1} + nh^k \geq 0.a.e.$$

Proof: Suppose C is (n,k) quasi class Q. Then for every $f \in L^2(\mu)$

$$\left\langle (C^{*k}C^{*(1+n)}C^{(1+n)}C^k - (1+n)C^{*k}C^*CC^k + nC^{*k}C^k) f, f \right\rangle \geq 0$$

Let f = ψ_A with $\mu(A) < \infty$

$$\left\langle (C^{*k}C^{*(1+n)}C^{(1+n)}C^k - (1+n)C^{*k}C^*CC^k + nC^{*k}C^k)\psi_A, \psi_A \right\rangle \geq 0$$

$$\int (h^{k+n+1}\psi_A - (1+n)h^{k+1}\psi_A + nh^k\psi_A) d\mu \geq 0$$

$$\Leftrightarrow \int_A (h^{k+n+1} - (1+n)h^{k+1} + nh^k) d\mu \geq 0$$

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$\Leftrightarrow h^{k+n+1} - (1+n)h^{k+1} + nh^k \geq 0$

Lemma 2.7 An operator $T \in B(H)$ is (n,k) – quasi - * - paranormal if and only if

$$T^* T^{*(n+1)} T^{(1+n)} T^k - (1+n) T^{*k} T T^* T^k + n T^{*k} T^k \geq 0$$

Theorem 2.8

Let C be a Composition operator on $L^2(\mu)$. Subsequently, C is (n,k) quasi -* - class Q iff $h^{k+n+1} - (1+n) h_k . E(h) o T^{-k} + nh^k \geq 0$

Proof: Observe

$$C^{*k} C C^* C^k f = C^{*k} C C^* (f o T^K)$$
$$= C^{*k} C (h . E(f o T^k) o T^{-1})$$
$$= C^{*K} c (h o T^{-1} . f o T^{k-1})$$
$$= C^{*k} (h o T^{-1.} . f o T^k) o T$$
$$= h_k . E(h) o T^{-k} . f$$

C is (n,k) quasi - * - class Q iff for every $f \in L^2(\mu)$

$$\left\langle \left(C^{*k} C^{*1+n} C^{1+n} C^k - (1+n) C^{*k} C C^* C^k + n C^{*k} C^k \right) f, f \right\rangle \geq 0$$

$$\Leftrightarrow h^{k+n+1} - (1+n) h_k . E(h) o T^{-k} + nh^k \geq 0$$

Example 2.9 Observe the space $l^2(\mathrm{N}, 2^N, \mu)(\omega)$ where $\omega = \langle m_n \rangle_{n=1}^{\infty}$ is a series of Positive real numbers. $\mu(n) = m_n$. Let $T : \mathrm{N} \to \mathrm{N}$ be a non – singular measurable Transformation. Then $T^n is$ also a non - singular measurable transformation for $n \in \mathrm{N}$.

$$h_k(s) = \frac{1}{m_s} (\textstyle\sum_j \in T^{-k}(s) m_j)^k ,$$

$$E(f)_k = \frac{\sum_j \in T^{-1} T(k) f_j m_j}{\sum_j \in T^{-1} T(k) m_j}$$

For all non- negative sequence f = $\langle f_n \rangle_{n=1}^{\infty}$ and $s, k \in \mathrm{N}$, By Therem 1, C is (n,k) – quasi class Q if and only if

$$\frac{1}{m_s^{\ n+k+1}} (\textstyle\sum_j \in T^{-1}(s) m_j)^{h+k+1} - (1+n) \frac{1}{m_s^{\ k+1}} (\textstyle\sum_j \in T^{-1}(s) m_j)^{k+1} + n$$
$$\frac{1}{m_s^{\ k}} (\textstyle\sum_j \in T^{-`}(s) m_j)^k \geq 0$$

for all $f = \langle f_n \rangle_{n=1}^{\infty}$ and $s, k \in \mathrm{N}$

By Theorem 2, C is (n,k) quasi - * _ class Q if and only if

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$$\frac{1}{m_s^{n+k+1}}(\sum_{j\in T^{-1}(s)m_j})^{n+k+1}-(1+n)\frac{1}{m_s^{k+1}}(\sum_{j\in T^{-1}(s)}m_j\frac{1}{m_{T^{-k}(s)}}\sum_{j\in T^{-(k+1)}}(s)m_j+$$

$$n\frac{1}{m_s^{k}}(\sum_{j\in T^{-1}(s)}m_j)^k\geq 0$$

Weighted Composition Operators

Let W be the weighted composition operator on $L^2(\mu)$.Let W^* be its adjoint which is stated by $W^*f=h.E(u.f)oT^{-1}$ for $f\in L^2(\mu)$. For a positive integer n, $u_n=u.(uoT).(uOT)^2.......(uoT)^{(n-1)}$.For $f\in L^2(\mu)$, $W^nf=u_n.foT^{-n}$

and $W^{*n}f=h_n.E(u_{n..}f)oT^{-n}$

Proposition 3.1 For $u\geq 0$

1.$W^*Wf=hE[(u^2)]oT^{-1}f$

2.$WW^*f=u(hoT)E(uf)$

Theorem 3.2 Let W be a weighted Composition operator on $L^2(\mu)$.Then W is (n,k) – quasi class Q iff $h_{k+n+1}.E(u_{k+n+1}^2)oT^{-(n+1)}-(1+n)h_{k+1}.E(u_{k+1}^2)oT^{-1}+nh_k.E(u_k^2)\geq 0$

<u>Proof:</u> Assume W is (n,k) – class Q .Then for $f\in L^2(\mu)$,

$$\langle(W^{*k}W^{*(1+n)}W^{(1+n)}W^k-(1+n)W^{*k}W^*WW^k+nW^{*k}W^k)f,f\rangle\geq 0$$

Let $f=\mu_A$ along with $\mu(A)\prec\infty$.Then

$$\langle(W^{*k}W^{*(1+n)}W^{(1+n)}W^k-(1+n)W^{*k}W^*WW^k+nW^{*k}W^k)\psi_A,\psi_A\rangle\geq 0$$

$$\Leftrightarrow\langle((h_{k+n+1.}E(u_{k+n+1}^2)T^{-(n+k+1)}-(1+n)h_{k+1}.E(u_{k+1}^2)oT^{-(k+1)}+nh_k.E(u_k^2)oT^{-k})\psi_A,\psi_A\rangle\geq 0$$

$$\Leftrightarrow\int(h_{k+n+1}.E(u_{k+n+1}^2)oT^{-(n+k+1)}-(1+n)h_{k+1}.E(u_{k+1}^2)oT^{-(k+1)}+nh_k.E(u_k^2)oT^{-k})\psi_Ad\mu\geq 0$$

$$\Leftrightarrow\int_A h_{k+n+1}.E(u_{k+n+1}^2)oT^{-(n+k+1)}-(1+n)h_{k+1}E(u_{k+1}^2)oT^{-(k+1)}+nh_k.E(u_k^2)oT^{-k}d\mu\geq 0$$

$$\Leftrightarrow h^{k+n+1}.E(u_{k+n+1}^2)oT^{-(n+1)}-(1+n)h_{k+1}E(u_{k+1}^2)oT^{-1}+nh_k.E(u_k^2)\geq 0$$

<u>Corollary 3.3</u> If W be a weighted composition operator on $L^2(\mu)$ and $T^{-1}(\mathrm{A})=\mathrm{A}$. In that case

W is (n,k) quasi class Q iff

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$$h_{k+n+1}.E(u^2_{_{k+n+1}})oT^{-(n+1)} - (1+n)h_k.hoT^{-(k+1)}.E(u^2_{k+1}) + nh_{k.}E(u^2_k) \geq 0 a.e.$$

Theorem 3.4 Let W be a weighted Composition operator on $L^2(\mu)$. Then W is (n,k) – quasi -* - class Q iff

$$h_{k+n+1}.E(u^2_{_{k+n+1}})oT^{-(n+1)} - (1+n)h_k.hoT^{-(k+1)}.E(u^2_{k+1}) + nh_{k.}E(u^2_k) \geq 0 a.e.$$

<u>Proof</u>: Assume W is (n,k) – quasi - * - class Q. Then for $f \in L^2(\mu)$

$$\left\langle \left(W^{*k}W^{*(1+n)}W^{(1+n)}W^k - (1+n)W^{*k}WW^*W^k + nW^{*k}W^k\right)f, f\right\rangle \geq 0$$

Let $f = \psi_A$ with $\mu(A) \prec \infty$.Then

$$\left\langle \left(W^{*k}W^{*(1+n)}W^{(1+n)}W^k - (1+n)W^{*k}WW^*W^k + nW^{*k}W^k\right)\psi_{A,}\psi_A\right\rangle \geq 0$$

$$\Leftrightarrow \left\langle \left(\left(h_{k+n+1.}E(u^2_{k+n+1}\right)oT^{-(n+k+1)} - (1+n)h_k.E(u_{k+1}(hoT)E(u_{k+1}))oT^{-k} + nh_k.E(u^2_k)oT^{-k}\right)\psi_A, \psi_A\right\rangle \geq 0$$

$$\Leftrightarrow \int (h_{k+n+1.}E(u^2_{k+n+1})oT^{-(n+k+1)} - (1+n)h_k.E(u_{k+1}(hoT)E(u_{k+1}(hoT)E(u_{k+1})oT^{-k} + nh_k.E(u^2_k)oT^{-k})\psi_A d\mu \geq 0$$

$$\Leftrightarrow \int_A h_{k+n+1}.E(u^2_{k+n+1})oT^{-(n+k+1)} - (1+n)h_k.E(u_{k+1}(hoT)E(u_{k+1})oT^{-k} + nh_k.E(u^2_k)oT^{-k}d\mu \geq 0$$

$$\Leftrightarrow h_{k+n+1}.E(u^2_{k+n+1})oT^{-(n+1)} - (1+n)h_k.hoT^{-(k-1)}.E(u^2_{k+1}) + nh_k.E(^2_k) \geq 0$$

<u>Corollary 3.5</u> Let W be a weighted Composition operator on $L^2(\mu)$ and $T^{-1}(\mathrm{A}) = \mathrm{A}$.
Then W is (n,k) – quasi - * - class Q iff

$$h_{k+n+1}.u^2_{k+n+1}oT^{-(n+1)} - (1+n)h_k.hoT^{-(k-1)}.u^2_{k+1} + nh_k.u^2_k \geq 0 a.e$$

The Operator transform $\overline{T} = |T|^{\frac{1}{2}}U|T|^{\frac{1}{2}}$ which is initiated in [1] by Aluthge is the Aluthge transform of T.

For a Composition operator C, the polar decomposition is given by $c = U|C|$ where $|c|f = \sqrt{h}f$ and $Uf = \frac{1}{\sqrt{hoT}} foT.$ In [2] Lambert stated general Aluthge transformation for Composition operator as $C_r = |C|^r U|C|^{1-r}$ and $C_r f = \left(\frac{h}{hoT}\right)^{\frac{r}{2}} foT$

That is C_r is the weighted composition operators with $\pi = \left(\frac{h}{hoT}\right)^{\frac{r}{2}}$ where o<r<1

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Since C_r is weighted composition operator it is easy to demonstrate

$|C_r| f = \sqrt{h[E(\pi)^2 oT^{-1}]} f$ and $|C_r^*| f = VE[Vf]$ where

$V = \frac{\pi\sqrt{hoT}}{[E(\pi\sqrt{hoT})^2]^{\frac{1}{4}}}$ Also we have

1. $C_r^k f = \pi_k(foT^k)$
2. $C_r^{*k} = h^k(E_{\pi k} f)oT^{-k}$
3. $C_r^{*k} C_r^k f = h^k(E\pi_k^2)oT^{-k} f$

Corollary 3.6 Let $C_r \in L^2(\mu)$ .Then C_r is (n,k) quasi class Q if and only if

$$h_{k+n+1}.E(\pi_{k+n+1}^2)oT^{-(n+1)} - (1+n)h_{k+1}E(\pi_{k+1}^2)oT^{-1} + nh_k.E(\pi_k{}^2) \geq 0 a.e$$

Proof: Since C_r is the weighted composition operator with $\pi = \left(\frac{h}{hoT}\right)^{\frac{r}{2}}$

It pursues from theorem 1 that C_r is(n,k) – quasi class Q if and only if

$$h_{k+n+1}.E(\pi_{k+n+1}^2)oT^{-(n+1)} - (1+n)h_{k+1}E(\pi_{k+1}^2)oT^{-1} + nh_k.E(\pi_k{}^2) \geq 0 a.e$$

Corollary 3.7 Let $C_r \in L^2(\mu)$ .Then W is (n,k) – quasi - * - class Q if and only if

$$h_{k+n+1}.E(\pi_{k+n+1}^2)oT^{-(n+1)} - (1+n)h_k.hoT^{-(k-1)}E(\pi_{k+1}^2) + nh_k.E(\pi_k^2) \geq 0 a.e$$

4 Conclusion:

In this study , some properties of A(n,k) class operators and (n,k) quasi class Q composition operators on varied space are identified

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