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Properties of Skew Normal Operators

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Abstract: *This paper deals with different ways of operant on a Hilbert space H(T) called skew n typical operant. A Skew n typical operant (or skew n normal operant) is a generalization of n typical operant. In this paper some basic properties of Skew n typical operant are characterized.*

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Introduction

The branch which deals with linear operators on function spaces is stated as operator theory. The study of functional analysis relies on the topology of function spaces.
Let L(H(T)) be the algebra of all bounded linear operant acting-upon H(T) is Hilbert space. Anuradhagupta [7] analyzed and introduced a new class of operator skew n typical operants. Skew n typical operants is a generalization of n typical operants.
In this paper some basic properties of skew n typical operants and skew n bitypical operants are characterized.

Skew n typical operant

An operator $V \in L(H)$ is called n typical if $V^*V^n = V^nV^*$

Quasi n typical if $V(V^*V^n) = (V^*V^n)V$. An operator $V \in L(H)$ is skew n typical if $(V^nV^*)V = V(V^*V^n)$ i.e., V commutes with n typical operants and it is denoted by $[snN]$

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Principle 2.1 Let $V \in L(H)$ be skew n typical operant which is distinctively homogeneous to an operator S iff the multiplicative commutative property of u and v is satisfied. Then S is skew n typical.

Proof:

$\because$ V is distinctively homogeneous to S, there is unitary operator U such that

$S = U^*VU$ that signifies $S^* = U^*V^*U$.

. it should be exhibited that $(S^nS^*)S = S(S^*S^n)$ .Since V is skew – n – typical.

Then $(V^nV^*)V = V(V^*V^n)$

Now

$$(S^nS^*)S = S(S^*S^n)$$
$$((U^*VU)^n(U^*VU)^*)U^*VU = U^*VU((U^*VU)^*(U^*VU)^n)$$
$$(U^*V^nUU^*V^*U)U^*VU = U^*VU(U^*V^*UU^*V^nU)$$
$$(U^*V^nV^*U)U^*UV = U^*UV(U^*V^*V^nU)$$
$$(V^nU^*UV^*)V = V(V^*U^*UV^n)$$
$$(V^nV^*)V = V(V^*V^n) \because VU^* = U^*V, V^nU^* = U^*V^n.$$
$$\therefore Sisskew-n-typical.$$

Example 2.2

Let S = $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$ and V = $\begin{pmatrix} 0 & 0 \\ 1 & 3 \end{pmatrix} onR^2$ .Then S+T is not skew typical.

Principle :2.3

Let $V = U|V|$ be the symmetric positive operant V. Then V = $U|V|$ is skew – n – typical. if $U|V| = |V|U$

Proof:

Assume $U|V|^n = |V|^n U, then$

$$(V^nV^*)V - V(V^*V^n) = (U|V|^n|V|U^*)U|V| - U|V|(|V|U^*U|V|^n)$$

$$= (U|V|^{n+1}U^*)U|V| - U|V|(|V|^{n+1})$$

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$= (|V|^{n+1}UU^*)U|V| - U|V|(|V|^{n+1})$

$= |V|^{n+1}\left(U|V| - U|V|\right)$

= 0

So V is Skew – n – typical.

Principle 2.4:

If V is a involution operant, then V^{-1} is also a skew -n – typical operant.

Proof:

$\because$ V is involution operant .$V^* = V$

Assume that V^{-1} is involution operant. Then $(V^{-1})^* = (V^*)^{-1} = V^{-1}$

To prove V^{-1} is Skew – n – typical.

Consider

$(V^nV^*)V = V(V^*V^n)$

$((V^{-1})^n(V^{-1})^*)V^{-1} = (V^{-1})^nV^{-1})V^{-1} = (V^{-1})^{n+2}$..............(1)

$V(V^*V^n) = V^{-1}((V^{-1})^*(V^{-1})^n) = (V^{-1})(V^*)^{-1}(V^{-1})^n = (V^{-1})(V^{-1})(V^{-1})^n = V^{-(n+2)}$..............(2)

$\therefore V^{-1}$ is skew – n – typical.

.

Principle2.5

Let V be an involution operant on a Hilbert space on H(T), then S^*VS is skew – n – typical.

Proof:

$\because$ V is involution operation $V^* = V$

Consider $(S^*V^nS)^* = S^*V^{n*}S^{**} = S^*V^nS = S^*V^nS$ ……………..(1)

$\Rightarrow S^*V^nS$ is involution operant. Then S^*V^nS is skew- n – typical.

Assume that S^*V^nS is involution operant.

consider $(S^*V^nS)(S^*VS)^*(S^*VS) = (S^*V^nS)(S^*VS)(S^*VS)$

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$= S^* V^{n+2} S$

$= (S^* VS)^{n+2}$ ………….(2)

$V(V^* V^n) = (S^* VS)(S^* VS)^* (S^* VS)^n = S^* VS(S^* VS) S^* V^n S = S^* V^2 V^n S = S^* V^{n+2} S = (S^* VS)^{n+2}$ ....(2)

$\therefore S^* VS$ is skew – n – typical operant .

Skew – n – bitypical operator

An Operator $V \in L(H(T))$ is skew – n – bitypical if $V(V^* V^n V^n V^*) = (V^n V^* V^* V^n)V$ i.e., V commutes with n bitypical operant and it is denoted by $[snBN]$

Principle 3.1

If $V \in [snBN] then,$

i) $\alpha V for any rea \ln umber \alpha$

ii) The diminution V/M of V to any concurrence space M of H that diminishes V.

Proof:

i) If α is any scalar, which is real then,

$(\alpha V)^* = \alpha V^*$

$\left[(\alpha V)^* (\alpha V)^n (\alpha V)^n (\alpha V)^* (\alpha V)\right] = \alpha V^* \alpha^n V^n \alpha^n V^n \alpha V^* \alpha V = \alpha^3 \alpha^{2n} V^{*2} V^{2n} V$

$= \alpha^{2n+3} V^{*2} V^{2n+1}$............(1)

$(\alpha V)\left[(\alpha V)^n (\alpha V)^* (\alpha V)^* (\alpha V)^n\right] = \alpha V \alpha^n V^n \alpha V^* \alpha V^* \alpha^n V^n = \alpha^3 \alpha^{2n} V V^{*2} V^{2n}$

$= \alpha^{2n+3} V^{*2} V^{2n+1}$..........(2)

From (1) &(2) αV is also skew – n- bitypical.

ii) If V is Skew – n- bitypical ,then $(V^* V^n V^n V^*)V = V(V^n V^* V^* V^n)$

Consider

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$$(V/M)^*(V/M)^n(V/M)^n(V/M)^*(V/M) = (V^*V^nV^nV^{*/}/M)(V/M)$$

$$(V/M)(V/M)^n(V/M)^*(V/M)^*(V/M)^n = (V/M)(V^n/M)(V^*/M)((V^*/M)(V^n/M)$$

$=V(V^nV^*V^*V^n)/M$

Hence V/M $\in [snBN]$

Principle 3.2

If $V \in L(H(T))$ is n bitypical then $V \in [SnBN]$.

Proof:

If $v \in L(H(T))$ is n bitypical then $V^*V^nV^nV^* = V^nV^*V^*V^n$

$(V^*V^n.V^nV^*)V = V^*V^nV^nV^*V$

$= V^nV^*V^*V^nV$

$=V^nV^*V^nV^*V$

$= V^nV^nV^*V^*V$

$=V(V^nV^*V^*V^n)$

Hence V is skew – n – bitypical.

Principle :3.3

If $V \in [SnBN]$and skew- n – bitypical operant. .If V and E are doubly commuting then VE is skew – n – bitypical.

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Proof:

Given $V \in [SnBN]$ and by definition of skew – n – bitypical operator. We know that

$V^*V^nV^nV^*)V = V(V^nV^*V^*V^n)$

Also $E \in [SnBN]$ so, $(E^*E^n.E^nE^*)E = E(E^nE^*.E^*E^n)$

$$((VE)^*(VE)^n(VE)^n(VE)^*)(VE) = (VE)^*(VE)^n(VE)^n(VE)^*(VE)$$

$= E^*V^*V^nE^nV^nE^nE^*V^*VE$

$= V^*E^*V^nE^nE^nV^nE^*V^*EV\ \ T$

$= V^*V^nE^*E^nE^nE^*V^nEV^*V$

$= V^*V^nE^*E^nE^nE^*EV^nV^*V$

$= \quad V^*V^n(E^*E^nE^nE^*)EV^nV^*V$

$= V^*V^nE(E^*E^nE^nE^*)V^nV^*V$

$= V^*V^n(E^*E^nE^nE^*)EV^nV^*V$

$= V^*V^nE(E^nE^*E^*E^n)V^nV^*V$

$= V^*V^nEE^nE^*E^*E^nV^nV^*V$

$= V^*EV^nE^nE^*E^*V^nE^nV^*V$

$= EV^*E^nV^nE^*V^nE^*V^*E^nV$

$= EE^nV^*V^nE^*V^nV^*E^*VE^n$

$= \quad EE^nV^*E^*V^nV^nV^*VE^*E^n$

$= EE^nV^*E^*V^nV^nV^*VE^*E^n$

$= EE^nE^*(V^*V^nV^nV^*)VE^*E^n$

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$= EE^n E^* V(V^n V^* V^* V^n)E^* E^n$

$= EE^n E^* VV^n V^* V^* V^n E^* E^n$

$= EE^n VE^* V^n V^* V^* E^* V^n E^n$

$= EVE^n E^* V^n V^* V^* E^* V^n E^n$

$= EVE^n V^n E^* V^* V^* V^* E^* V^n E^n$

$= EVE^n V^n V^* E^* V^* E^* E^n V^n$

$= (EV)\left[(EV)^n (EV)^* (EV)^* (EV)^n\right]$

Hence EV is skew n – bitypical operant.

Principle:2.6

Let V = $\cup|V|$ be the symmetric positive of an operant V. Then V= $\cup|V|$ is skew n bitypical if $\cup|V| = |V|\cup$, $(U|V|)^* = |V|U^*$

Proof:

Consider

$(V^* V^n V^n V^*)V - V(V^n V^* V^* V^n) =$

$\left(|V|\cup^* \cup|V|^n \cup|V|^n |V|\cup^*\right)\cup\left||V| - \cup|V|\right|\left(\cup|V|^n |V|\cup^* |V|\cup^* \cup|V|^n\right)$

$= \left(|V|^{n+1} \cup|V|^{n+1}|V| - \cup|V|\left(|V|^n \cup\cup^* \middle|V|V\|V|^n\right|\right))$

$= |V|^{n+1} \cup V^{n+1}|V| - \cup\left|V\left(|V|^{n+1}|V|^{n+1}\right)\right|$

= 0

Hence Vis skew n bitypical.

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Principle 3.7

Let $V \in L(H(T))$ be skew n bitypical operant which is unique similar to an operator L if and only if $V\cup = \cup V, V\cup^{*} = \cup^{*} V, V^{*}\cup = \cup V^{*}$,then L is skew n – bitypical.

Proof:

V is unitarily analogous to L, there is a unitary operator $\cup$ such that $L = \cup^{*} V \cup$

That sigifies $L^{*^{*}} = \cup^{*} V^{*} \cup$ .we prove that $\left(L^{*} L^{n} L^{n} L^{*}\right) L = L(L^{n} L^{*} L^{*} L^{n})$.

Since V is skew – n – bitypical then $(V^{*} V^{n} V^{n} V^{*}) V = V(V^{n} V^{*} V^{*} V^{n})$

Consider $(L^{*} L^{n} L^{n} L^{*}) L = (\cup^{*} V^{*} \cup \cup^{*} V^{n} \cup \cup^{*} V^{n} \cup \cup^{*} V^{*} \cup) \cup^{*} V \cup$

$= \cup^{*} V^{*} V^{n} V^{n} V^{*} V \cup$

$= \cup^{*} V^{*} V^{n} V^{n} V^{*} \cup V$

$= \cup^{*} V^{*} V^{n} V^{n} \cup V^{*} V$

$= \cup^{*} V^{*} V^{n} \cup V^{n} V^{*} V$

$= \cup^{*} V^{*} \cup V^{n} V^{n} V^{*} V$

$= U^{*} \cup V^{*} V^{n} V^{n} V^{*} V$

$= V^{*} V^{n} V^{n} V^{*} V$

$= (V^{*} V^{n} V^{n} V^{*}) V$ ....(1)

$L(L^{n} L^{*} L^{*} L^{n}) = \cup^{*} V \cup (\cup^{*} V^{n} \cup \cup^{*} V^{*} \cup \cup^{*} V^{*} \cup \cup^{*} V^{n} \cup)$

$= \cup^{*} V V^{n} V^{*} V^{*} v^{n} \cup$

$= \cup^{*} V V^{n} V^{*} V^{*} \cup V^{n}$

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$= \bigcup^{*} VV^{n}V^{*} \bigcup V^{*}v^{n}$

$= \bigcup^{*} VV^{n} \bigcup V^{*}V^{*}V^{n}$

$= \bigcup^{*} V \bigcup V^{n}V^{*}V^{*}V^{n}$

$= \bigcup^{*} \bigcup VV^{n}V^{*}V^{*}V^{n}$

$= VV^{n}V^{*}V^{*}V^{n}$

$= V(V^{n}V^{*}V^{*}V^{n})$ ………………(2)

From (1) and (2) L is skew n bitypical operant.

Involution operation and skew n bitypical operant

Principle 4.1

If the operator V is involution operant and also skew – n- bitypical operant,then

V* is also skew - n - bitypical operant.

Proof:

Since V is skew – n – bitypical operant, we have,

$(V^{*}V^{n}V^{n}V^{*})V = V(V^{n}V^{*}V^{*}V^{n})$

$\because$ V is self adjoint we have $V^{*} = V$

Replace $V^{*} = V$

$(V^{*}V^{n}V^{n}V^{*})V = V^{*}V^{n}V^{n}V^{*}V$

$= VV^{*n}V^{*n}VV^{*}$

$= V^{2}V^{*2n+1}$…………………..(1)

$V(V^{n}V^{*}V^{*}V^{n}) = VV^{n}V^{*}V^{*}V^{n}$

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$= V^{*} V^{*n} V V V^{*n}$

$= V^{2} V^{*2n+1}$ ……………………..(2)

From (1) and (2) V^{*} is skew –n – bitypical operant.

Principle :4.2

If V is involution operator then V^{-1} is also a skew – n – bitypical operant..

Proof:

By the definition of involution operant we have $V^{*} = V$

Assume that V^{-1} involution operant . $\therefore (V^{-1})^{*} = V^{-1}$

To prove that V^{-1} is skew – n – bitypical operant

Consider,

$(V^{*} V^{n} V^{n} V^{*}) V = (V^{-1})^{*} (V^{-1})^{n} (V^{-1})^{n} (V^{-1})^{*} V^{-1}$

$= V^{-1} (V^{-1})^{n} (V^{-1})^{n} V^{-1} V^{-1}$

$= (V^{-1})^{3} (V^{-1})^{2n} = (V^{-1})^{2n+3}$ ………………….(1)

$V (V^{n} V^{*} V^{*} v^{n}) = (V^{-1})(v^{-1})^{n} (v^{-1})^{*} (v^{-1})^{*} (V^{-1})^{n}$

$= (V^{-1})(V^{-1})^{n} (V^{-1})^{*} (V^{-1})^{*} (v^{-1})^{n}$

$= (V^{-1})^{2n+3}$ ……………………………(2)

From (1) & (2), V^{-1} is skew – n – bitypical operant.

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Principle 4.3:

If V is a involution operant on a Hilbert space H(T), then $L^{*}VL$

Is skew – n – bitypical.

Proof:

$\because$ V is involution operant, $V^{*} = V$

Consider, $(L^{*}VL)^{*} = L^{*}V^{*}L$ …………..(1) $\therefore V^{*} = V \Rightarrow L^{*}VL$ is self adjoint

By Principle (3.2) if $L^{*}VL$ is involution operant, then it is skew – n – bitypical.

Assume $L^{*}VL$ is involution operant..

consider $(V^{*}V^{n}V^{n}V^{*}V = (L^{*}VL)^{*}(L^{*}VL)^{n}(L^{*}VL)^{n}(L^{*}VL)^{*}(L^{*}VL)$

$= (L^{*}V^{*}L)(L^{*}V^{n}L)(L^{*}V^{n}L)(L^{*}V^{*}L)(L^{*}VL)$

$= (L^{*}VL)(L^{*}V^{n}L)(L^{*}V^{n}L)(L^{*}vL)(L^{*}VL)$

$= (L^{*}V^{n}L)^{2}(L^{*}VL)^{3}$ ………………….(1)

$V(V^{n}V^{*}V^{*}v^{n}) = (L^{*}VL)(L^{*}VL)^{n}(L^{*}VL)^{*}(L^{*}VL)^{*}(L^{*}VL)^{n}$

$= (L^{*}VL)(L^{*}V^{n}L)(L^{*}V^{*}L)(L^{*}V^{*}L)(L^{*}V^{n}L)$

$= (L^{*}VL)(L^{*}V^{n}L)(L^{*}VL)(L^{*}VL)(L^{*}V^{n}L)$

$= (L^{*}VL)^{3}(L^{*}V^{n}L)^{2}$ …………………………(2)

=From (1) &(2) $L^{*}VL$ is skew – n – bitypical

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Quasi (n,m) power D Hypotypical operators

An operator $V \in B(H)^D$ is said to be Quasi (n,m) power D- hypotypical if
$V^*(V^{*m}(V^D)^n - (V^D)^n V^{*m})V \geq 0$ for m,n = 1,2,….

Proposition 5.1

If V.W $\in B(H)^D$ are unitarily equivalent and if V is quasi (n,m) power – D-hypotypical operant. Then so is W.

Proof:

Let V be an quasi (n,m) power D –hypotypical operant and W be unique similar of V.
Then there exists unitary operant U such that $V = UWU^*$ so $V^n = UW^nU^*$

Consider

$$
\begin{aligned}
&V^*(V^{*m}(V^D)^n V = (== UWU^*)^*(UW^mU^*)^*(UW^DU^*)^n(UWU^*) \\
&= UW^*U^*UW^{*m}U^*U(W^D)^nU^*UWU^* \\
&= UW^*W^{*m}(W^D)^nWU^* \\
&- \geq UW^*(W^D)^nWU^* \\
&= UW^*U^*U(W^D)^nU^*UW^{*m}WU^* \\
&= V^*(V^D)^nUW^{*m}U^*W \\
&= V^*(V^D)^nV^{*m}UU^*W \\
&= V^*(V^D)^nV^{*m}UWU^* \\
&== V^*(V^D)^nV^{*m}V \\
&i.e V^*(V^{*m}(V^D)^m - (V^D)^nV^{*m})V \geq 0
\end{aligned}
$$

Principle 5.2

Let $V_{1,}V_{2,}.....V_k$ are quasi (n,m) power D hypotypical operant in $B(H)^D$.Then
$(V_1 \oplus V_2 \oplus .. \oplus .V_k$ is

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Quasi (n,m) power D hypotypical operant and $V_1 \otimes V_2 \otimes .... \otimes V_k$ is quasi(n,m) power D – hypotypical operant.

Proof:

Consider

$$(V_1 \oplus V_2 \oplus ...... \oplus V_k)^*((V_2 \oplus V_2 \oplus ... \oplus V_k)^*)^m((V_1 \oplus V_2 \oplus .... \oplus V_k)^D)^n(V_1 \oplus V_2 \oplus ..... \oplus V_k) =$$

=

$$\begin{aligned}
&(V_1^* \oplus V_2^* \oplus ...... \oplus V_k^*)(V_1^{*m} \oplus V_2^{*m} \oplus ... \oplus V_k^{*m})((V_1^D \oplus (V)^n \oplus (V_2^D)^n \oplus ,,,, \oplus (V_k^D)^n (V_1 \oplus V_2 \oplus .... V_k) \\
&= (V_1^* V_1^{*m} \oplus V_2^* V_2^{*m} \oplus .... \oplus V_k^* V_k^{*m})((V_1^D)^n V_1 \oplus (V_2^D)^n V_2 \oplus .... \oplus (V_K^D)^n V_k) \\
&= V_1^* V_1^{*m} (V_1^D)^n V_1 \oplus V_2^* V_2^{*m} (V_2^D)^n V_2 \oplus .... \oplus V_k^* V_k^{*m} V_k^D V_k \\
&= (V_1 \oplus V_2 \oplus ..... \oplus V_k)^*((V_1 \oplus V_2 \oplus ..... \oplus V_k)^D)^n((V_1 \oplus V_2 \oplus ...... \oplus V_k)^*)^m(V_1 \oplus V_2 \oplus ...... \oplus V_k) \\
&\geq V^*(V^D)^n(V^*)^m V \\
&Therefore (V_1 \oplus V_2 \oplus .... \oplus V_k)
\end{aligned}$$

is quasi (n,m) power D – hypotypical operant.

Consider

$$\begin{aligned}
&V^* V^{*m} (V^D)^n V = (V_1 \otimes V_2 \otimes .... \otimes V_k)^*((V_1 \otimes V_2 \otimes ..... \otimes V_k)^*)^m (V_1^D \otimes V_2^D \otimes ..... \otimes V_k^D)^n (V_1 \otimes V_2 \otimes .... \otimes V_k) \\
&(V_1^* \otimes V_2^* \otimes ...... \otimes V_k^*)(V_1^{*m} \otimes V_2^{*m} \otimes ... \otimes V_k^{*m})((V_1^D \otimes (V)^n \otimes (V_2^D)^n \otimes ,,,, \otimes (V_k^D)^n (V_1 \otimes V_2 \otimes ... \otimes .V_k) \\
&= (V_1^* V_1^{*m} \otimes V_2^* V_2^{*m} \otimes .... \otimes V_k^* V_k^{*m})((V_1^D)^n V_1 \otimes (V_2^D)^n V_2 \otimes .... \otimes (V_K^D)^n V_k) \\
&= V_1^* V_1^{*m} (V_1^D)^n V_1 \otimes V_2^* V_2^{*m} (V_2^D)^n V_2 \otimes .... \otimes V_k^* V_k^{*m} V_k^D V_k \\
&= (V_1 \otimes V_2 \otimes ..... \otimes V_k)^*((V_1 \otimes V_2 \otimes ..... \otimes V_k)^D)^n((V_1 \otimes V_2 \otimes ...... \otimes V_k)^*)^m(V_1 \otimes V_2 \otimes ...... \otimes V_k) \\
&\geq V^*(V^D)^n(V^*)^m V \\
&Therefore (V_1 \otimes V_2 .... \otimes V_k)
\end{aligned}$$

is quasi(n,m) power D – hypotypical operant.

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Quasi Square n Bitypical operators

An operator $V \in L(H(T))$ is called Quasi Square n Bitypical if

$$V(V^*)^2 V^{2n} V^{2n} (V^*)^2 = (V^{*2} V^{2n} V^{2n} (V^*)^2)V$$

Principle 6.1

If V is a quasi a square n – bitypical operant which is a self adjoint operant, then V^* is also quasi Square – n bitypical operant,

Proof:

If V is quasi Square – n – bitypical

$$V(V^*)^2 V^{2n} V^{2n} (V^*)^2 = (V^*)^2 V^{2n} V^{2n} (V^*)^2 V$$

operant,then $\because V is self adjoint.$

$$V^* = V$$

$$V^*((V^*)^*)^2 (V^*)^{2n} (V^*)^2 ((V^*)^*)^2 = V^*(V^2)(V^*)^{2n} V^2 \text{...........(1)}$$

$$Here V^* = V$$

$$(V^*)^2 V^{2n} V^{2n} (V^*)^2 V = (V)^2 V^{*2n} V^{*2n} (V)^2 V^* \text{...........................(2)}$$

From (1) and (2) V^* is also quasi square n – bitypical operatos.

4.Conclusion

In this paper some of the properties of skew n normal operants and skew-n-bitypical operants were studied. With the view of upcoming researches this class of operant can be developed to higher class of operant .

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