PERIODICO di MINERALOGIA
ISSN: 0369-8963
Volume 91, No. 5, 2022

A Distance Measure Between Intuitionistic Fuzzy Multisets Sets and its Application in Medical Diagnosis

A.MAHESWARI[1]
Department of Mathematics, Velammal College of Engineering& Technology, Madurai, Tamil Nadu, India
S.KARTHIKEYAN[2]
Department of Mathematics, Velammal College of Engineering & Technology, Madurai, Tamil Nadu, India
P.VEERAMMAL[4]
Department of Mathematics, Saraswathi Narayanan College, Madurai, Tamil Nadu, India
M.PALANIVELRAJAN[3]
Department of Mathematics, Government Arts College, Melur Tamil Nadu, India

Abstract –

In this paper a new formula for finding the distance between Intuitionistic Fuzzy Multisets is proposed. The efficiency and advantages of the new distance are explained by giving examples in medical diagnosis and pattern recognition and finally we compared the results with the existing results.

Keywords - *Fuzzy Sets, Intuitionistic Fuzzy Sets, Multisets, Intuitionistic Fuzzy Multisets.*

INTRODUCTION

In 1965 Lofti Zadeh [1] introduced Fuzzy sets where a membership function is assigned to each element of the universe of discourse, from the unit interval [0,1] to specify the degree of belongingness to the set under consideration. As a generalisation of Fuzzy Sets in 1983, Krassimir. T. Atanassov [2] introduced the concept of Intuitionistic Fuzzy sets (IFS) by assigning a degree for not belonging together with the degree of belonging of the fuzzy set. Based on the concept of multi set Yagar [3] introduced the concept of Fuzzy Multiset. By combining the concept of Fuzzy Multiset and IFS, Shinoj T. K, Sunil Jacob John [4] defined Intuitionistic Fuzzy MultiSets. The distance measures and the similarity measures of intuitionistic fuzzy sets have received much consideration in recent years due to their importance in decision making, medical diagnosis, pattern recognition etc. In last decade several formulae have been proposed to find the distance between Intuitionistic Fuzzy Sets.

As an extension of distance between Fuzzy Sets Eulalia Szmidt and Janusz Kacprzyk [5] proposed a method to find the distance between two Intuitionistic Fuzzy Sets by employing the geometric interpretation of IFS. In that paper they defined Hamming distance, Normalized Hamming distance, Euclidean distance and normalized Euclidean distance and compared the distances for Fuzzy Sets and IFS. Li Dengfeng and Cheng Chuntian [8] introduced the similarity measure between IFS and applied it in pattern recognition. Wang.W and Xin X [9] proposed some new distance measures by considering the distance measure as the duality of similarity measures and applied their method in pattern recognition. Jin Han Park , Ki Moon Lim and Young Chel

ISSN: 0369-8963

Kwun [10] introduced a new method for measuring the distance between IFS using the three dimensional representation of IFS and applied the same in pattern recognition. Supriya Kumar De, Ranjit Biswas, Akhil Ranjan Roy [11] applied IFS for medical diagnosis through Intuitionistic Fuzzy Relation. Eulalia Szmidt and Janusz Kacprzyk [12] obtained a solution from the smallest distance between symptoms and the patients by representing them as IFS in medical diagnosis. A.G. Hatzimichailidis, G.A. Papakostas, V.G. Kaburlasos [13] defined a matrix norm based distance on IFS and applied it in pattern recognition. P. A. Ejegwa, A. J. Akubo O. M. Joshua [14] used normalized Euclidean method in career determination by obtaining the smallest distance between each student and each career. Pramanik S, Mondal K.[21] proposed the concept of the tangent similarity measure of IFS and applied to decision making.

In this paper we propose a new formula for finding the distance between two Intuitionistic Fuzzy Multisets in which the degree of belonging, degree of non belonging and the degree of hesitancy are used and then the proposed formula is applied in medical diagnosis and in pattern recognition. This article is organized as follows. In section 2 some basic definitions of IFS and some existing distance measures are recalled. In section 3 the proposed method is introduced and in section 4 examples are given using the new formula.

PRELIMINARIES

In this section, we recall some relevant definitions and results. Throughout this paper, X is a universal set.

Definition 2.1 $A\ Fuzzy\ set\ (FS)\ A \in X\ is\ defined\ as\ an\ object\ of\ the\ form\ A = \{< x, \mu_A(x) > x \in X\} where\ the\ function\ \mu_A: X \to [0,1]\ denote\ the\ degree\ of\ membership\ function\ of\ A.$

Definition 2.2 An Fuzzy multiset (FMS) A ∈ X is defined as an object of the form A = { <x , $\left(\mu_A^1(x), \mu_A^2(x), \dots \dots \mu_A^n(x)\right)$ >/x∈ X }, where the function $\mu_A^i(x)$: X→ [0,1] , $1 \leq i \leq$ n with$\mu_A^1(x) \geq\ \mu_A^2(x) \geq \dots \dots \geq \mu_A^n(x)$.

Definition 2.3 An Intuitionistic Fuzzy Set (IFS) A∈ X is defined as an object of the form A = { <x , $\mu_A(x)$, $\nu_A\ (x)$ >/x∈X }, where the function μ_A: X→[0,1] and ν_A: X→[0,1]denote the degree of belongingness and the degree of non-belongingness of A respectively and $0 \leq \mu_A(x)$ + $\nu_A(x) \leq 1$, for every x in X. To measure hesitancy degree of an element to an IFS, Atanassov introduced a function given by: $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, ∀ x∈X and $0 \leq \pi_A(x) \leq 1$

Definition 2.4 An Intuitionistic Fuzzy Multiset (IFMS) A∈ X is defined as an object of the form A = { <x , $\left(\mu_A^1(x), \mu_A^2(x), \dots \dots \mu_A^n(x)\right)$, $\left(\nu_A^1(x), \nu_A^2(x), \dots \dots \nu_A^n(x)\right)$ >/x∈ X }, where the function $\mu_A^i(x)$: X→ [0,1] , $1 \leq i \leq$ n with$\mu_A^1(x) \geq\ \mu_A^2(x) \geq \dots \dots \geq \mu_A^n(x)$and $\nu_A^i(x)$: X→ [0,1] , $1 \leq i \leq$ ndenote the degree of membership and the degree of non-membership function of A respectively and $0 \leq \mu_A^i(x)$+ $\nu_A^i(x) \leq 1$, for every x in X.

 https://doi.org/10.37896/pd91.5/91513

ISSN: 0369-8963

Remark: We arrange the membership sequence in decreasing order but the corresponding non-membership sequence need not be in decreasing order or increasing order.

Definition 2.5 An Intuitionistic Fuzzy Multisets (IFMSs) A, B ∈ X are defined as an object of the form

$$A = \{ <x , \left(\mu_A^1(x), \mu_A^2(x), \dots \dots \mu_A^n(x)\right) , \left(\nu_A^1(x), \nu_A^2(x), \dots \dots \nu_A^n(x)\right) > \big/ x \in X \}$$

and

$$B = \{ <x , \left(\mu_B^1(x), \mu_B^2(x), \dots \dots \mu_B^n(x)\right) , \left(\nu_B^1(x), \nu_B^2(x), \dots \dots \nu_B^n(x)\right) > \big/ x \in X \}$$

Definition 2.6 Let X=$\{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse. For any A and B ∈ IFSs(X), the operation is defined: $A = B$ iff $\mu_A = \mu_B$ and $\nu_A = \nu_B$.

Definition 2.7 A notion of distance d in a nonempty set X is a real valued function defined by d: X×X→[0, +∞), which satisfies the following conditions:
(1) $d(x, y) \geq 0$, for all x, y∈X,
(2) $d(x, y) = 0 \Leftrightarrow x = y$, for all x, y∈X,
(3) $d(x, y) = d(y, x)$, for all x, y∈X,
(4) $d(x, z) + d(z, y) \geq d(x, y)$, for all x, y, z ∈X.

Remark we briefly review some existing distance measures between IFSs as follows:

- **Szmidt and Kacprzyk's distance measures** [6]

$$d_H(A, B) = \frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\,\pi_A(x_i) - \pi_B(x_i)|)$$

$$d_E(A, B) = \sqrt{\frac{1}{2n}\sum_{i=1}^{n}[(|\mu_A(x_i) - \mu_B(x_i)|\,)^2 + (|\nu_A(x_i) - \nu_B(x_i)|)^2 + (|\,\pi_A(x_i) - \pi_B(x_i)|)^2]}$$

- **Grzegorzewski's distance measure [18]**

$$d_{nH}(A, B) = \frac{1}{n}\sum_{i=1}^{n}\max\{|\mu_A(x_i) - \mu_B(x_i)|\,, |\nu_A(x_i) - \nu_B(x_i)|\}$$

- **Yang and Francisco's distance measure [19]**

$$d_l(A, B) = \frac{1}{n}\sum_{i=1}^{n}\max\{|\mu_A(x_i) - \mu_B(x_i)|\,, |\nu_A(x_i) - \nu_B(x_i)|\,, |\pi_A(x_i) - \pi_B(x_i)|\}$$

- **Minxia Luo, Ruirui Zhao [16]**

$$d_f(A, B; f) \triangleq \frac{\|\,\Pi(\mu_A) - \Pi(\mu_B)\,\| + \|\,\Pi(\nu_A) - \Pi(\nu_B)\,\| + \|\,\Pi(\pi_A) - \Pi(\pi_B)\,\|}{3n}$$

 https://doi.org/10.37896/pd91.5/91513

A proposed new distance measure between intuitionistic fuzzy multisets

In this section, we introduce the concept of proposed new distance measure between intuitionistic fuzzy multisets and we give the notation of the proposed new distance measure between intuitionistic fuzzy multisets.

Definition 3.1 Given a finite universe of discourse X= $\{x_1, x_2, \ldots, x_n\}$. Let A, B be two intuitionistic fuzzy multisets in IFMS(X), f: [0, 1] × [0, 1] → [0, 1] a strictly increasing (or decreasing) binary function for each argument. A distance measure is a function

d_{pv} : IFMS(X) × IFMS(X) → [0, 1] defined for A, B ∈ IFMS(X) by

$$d_{pv}(A,B) = \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda$$

Where $\lambda \in [0, 1]$.

Clearly we can prove the above defined distance measure is a metric as follows.

i) $d_{pv}(A,B) \geq 0$

ii) suppose $d_{pv}(A,B) = 0$

$\Rightarrow \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda = 0$

$\Rightarrow \int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda = 0$

$\Rightarrow \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda = 0$

$\Rightarrow \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\} = 0$

$\Rightarrow \{|\mu_A(x_i) - \mu_B(x_i)|\} = 0$, $\{|\nu_A(x_i) - \nu_B(x_i)|\} = 0$ and $\{|\pi_A(x_i) - \pi_B(x_i)|\} = 0$

$\Rightarrow \mu_A(x_i) = \mu_B(x_i)$, $\nu_A(x_i) = \nu_B(x_i)$ and $\pi_A(x_i) = \pi_B(x_i)$

Thus A = B

that is $d_{pv}(A,B) = 0$ implies A = B

iii) $d_{pv}(A,B) = \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda$

$= \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_B(x_i) - \mu_A(x_i)| + |\nu_B(x_i) - \nu_A(x_i)|\,\lambda + |\pi_B(x_i) - \pi_A(x_i)|\lambda^2\}\, d\lambda$

$= d_{pv}(B,A)$

iv) $d_{pv}(A,C) = \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_C(x_i)|\,\lambda + |\pi_A(x_i) - \pi_C(x_i)|\lambda^2\}\, d\lambda$

$= \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i) + \mu_B(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_B(x_i) + \nu_B(x_i) -$ $\nu_C(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i) + \pi_B(x_i) - \pi_C(x_i)|\lambda^2\}\, d\lambda$

$\leq \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\mu_B(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\nu_B(x_i) - \nu_C(x_i)|\,\lambda +$ $|\pi_A(x_i) - \pi_B(x_i)| + |\pi_B(x_i) - \pi_C(x_i)|\lambda^2\}\, d\lambda$

$= \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda$

$+ \frac{1}{4}\int_0^1 \sum_{i=1}^{n} \{|\mu_B(x_i) - \mu_c(x_i)| + |\nu_B(x_i) - \nu_C(x_i)|\,\lambda + |\pi_B(x_i) - \pi_C(x_i)|\lambda^2\}\, d\lambda$

 https://doi.org/10.37896/pd91.5/91513

ISSN: 0369-8963

$= d_{pv}(A,B) + \ d_{pv}(B,C)$

Thus $d_{pv}(A,C) \leq d_{pv}(A,B) + \ d_{pv}(B,C)$

Application

In this section, we illustrate the superiority of the proposed new distance measure, and a comparison between the proposed new distance measure distance measure and some existing distance measures is made based on numerical comparisons through the following Numerical Example.

ALGORITHMS

Let X= $\{x_1, x_2, \ldots, x_n\}$be a finite universe of discourse, there exist m patients which are represented by IFMSs $P_j = \{ < x_i, \mu^i_{P_j}(x)\ ,\ \nu^i_{P_j}(x) > \big/ x_i \in \mathrm{X}\ \}$, and the corresponding test sample which is represented by an IFMSs $S_k = \{ < x_i\ ,\ \mu^i_{S_k}(x)\ ,\ \nu^i_{S_k}(x) > \big/ x_i \in \mathrm{X}\ \}$,where j, k = 1,2,…m .The process is as follows

Step 1

- Write a Table Q from the set of patients P_j to the set of symptoms S_k
  (i.e) $\mathrm{Q} = (P_j \to S_k\ )$
- Write a Table R from the set of symptoms S_k to the set of diagnoses D_t.
  (i.e) $\mathrm{R} = (S_k\ \to D_t)$

Step 2 Construct the Table Q and R by three numbers, namely μ- membership function, ν – non-membership function and π-hesitation function.

Step 3 Calculate the distance measure $d(\ P_j, D_t)$ between $P_j \to S_k$and $S_k\ \to D_t$

Step 4 Find

- Classification results (CS),
  CS = min $\{d\left(P_j, D_t\right) / \ j \neq t\ , for\ \ j, t$ = 1,2,…$m\}$
- Calculate degree of confidence (DC),
  DC = max$\sum_{\substack{j,t=1 \\ j\neq t}}^{m}\{\left|\ d(\ P_j, D_t) - CS\right|\ \}$, [15].

If the DC is greater, then the result of the specific distance metric is the more confident.

Example 4.2

Consider the three patterns P_1, P_2, P_3 and the test sample S, [8,13,16]. as presented in the following Table **4.2.1**

Table 4.2.1

	A_1	A_2	A_3
P_1	(1.0,0.0)	(0.8,0.0)	(0.7,0.1)
P_2	(0.9,0.1)	(1.0,0.0)	(0.9,0.0)
P_3	(0.6,0.2)	(0.8,0.0)	(1.0,0.0)
S	(0.5,0.3)	(0.6,0.2)	(0.8,0.1)

 https://doi.org/10.37896/pd91.5/91513

ISSN: 0369-8963

Each characteristic for the above P_i and $A_i$are described by three numbers μ, ν and π in the following **Table 4.2.2.**

Table 4.2.2

	Attributes $x_1 x_2 x_3$		
P_1	(1.0,0.0,0.0)	(0.8,0.0,0.2)	(0.7,0.1,0.2)
P_2	(0.9,0.1,0.0)	(1.0,0.0,0.0)	(0.9,0.0,0.1)
P_3	(0.6,0.2,0.2)	(0.8,0.0,0.2)	(1.0,0.0,0.0)
S	(0.5,0.3,0.2)	(0.6,0.2,0.2)	(0.8,0.1,0.1)

Let us consider **Table 4.2.2** where the element $P_i \to S$. The distance measure $d_{pv}(P_1, S)$between $P_1 \to S$ is obtained by Definition 3.1,

$$d_{pv}(P_1, S) = \frac{1}{4}\int_0^1 \sum_{i=1}^{3} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\,\lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda$$

$$d_{pv}(P_1, S) = \frac{1}{4}\int_0^1 [\{|\,1.0 - 0.5| + |\,0.0 - 0.3|\,\lambda + |0.0 - 0.2|\lambda^2\} + \{|\,0.8 - 0.6| + |\,0.0 - 0.2|\,\lambda + |0.2 - 0.2|\lambda^2\} + \{|\,0.7 - 0.8| + |\,0.1 - 0.1|\,\lambda + |0.2 - 0.1|\lambda^2\}]\, d\lambda$$

$$= \frac{1}{4}\int_0^1 [\{0.4 + 0.3\lambda + 0.2\lambda^2\} + \{0.2 + 0.2\,\lambda + 0.0\lambda^2\} + \{0.1 + 0.0\lambda + 0.1\lambda^2\}]\, d\lambda$$

$$= \frac{1}{4}\int_0^1 [0.7 + 0.5\lambda + 0.3\lambda^2]\, d\lambda$$

$$=0.2625$$

Similarly, calculate the remaining distance measures are calculated as:
$d_{pv}(P_2, S) = 0.3458$,$d_{pv}(P_3, S) = 0.1833$.

By using the **4.2. Algorithm**

Calcuate CS

$$\begin{aligned} \text{CS} &= \min\{d_{pv}(P_j, S) / for\ \ j = 1,2,3\} \\ &= \min\{d_{pv}(P_1, S), d_{pv}(P_2, S), d_{pv}(P_3, S)\} \\ &= \min\{0.2625, 0.3458, 0.1833\} \\ &= 0.1833 \\ &= d_{pv}(P_3, S) \end{aligned}$$

Calculate degree of confidence (DC),

$$\begin{aligned} \text{DC} &= \sum_{\substack{j,t=1 \\ j \neq t}}^{3} \{|\,d(P_j, S) - CS)|\,\} \\ &= \{|d_{pv}(P_1, S) - d_{pv}(P_3, S)| + |d_{pv}(P_2, S) - d_{pv}(P_3, S)|\} \\ &= \{|\,0.2625 - 0.1833)| + |\,0.3458 - 0.1833)|\} \\ &= \{0.0792 + 0.1625\} \\ &= 0.2417 \end{aligned}$$

We put all $d_{pv}(P_i, S)$, CS and DC values in**Table 4.2.3.**

https://doi.org/10.37896/pd91.5/91513

ISSN: 0369-8963

Table 4.2.3

Distance	distance measures			Results	
	$d_{pv}(P_1,S)$	$d_{pv}(P_2,S)$	$d_{pv}(P_3,S)$	CCS	DC
d_{pv}	0.26	0.35	**0.18**	P_3	0.24

We summarize the results of different distance measures [16] with **Table 4.2.4**.

Table 4.2.4

Distances	distance measures			Results	
	$d(P_1,S$	$d(P_2,S$	$d(P_3,S)$	CS	DC
d_T	0.32	0.19	0.15	P_3	0.21
d_R	0.19	0.17	0.10	P_3	0.16
d_L	0.11	0.08	0.04	P_3	0.11
d_{KD}	0.26	0.25	0.16	P_3	0.19
d_M	0.21	0.25	0.15	P_3	0.16
d_{LA}	0.21	0.27	0.15	P_3	0.18
d_G	0.41	0.24	0.19	P_3	0.27
d_H	0.27	0.30	0.17	P_3	0.23
d_E	0.28	0.29	0.16	P_3	0.25
d_{nH}	0.27	0.30	0.17	P_3	0.23
d_l	0.27	0.30	0.17	P_3	0.23
d_1	0.16	0.18	0.09	P_3	0.16
d_2^1	0.22	0.23	0.15	P_3	0.15
d_p	0.27	0.30	0.17	P_3	0.23
d_s	0.11	0.11	0.06	P_3	0.10
d_h	0.21	0.22	0.16	P_3	0.11
d_f	0.35	0.42	0.24	P_3	0.29
d_{pv}	0.26	0.35	0.18	P_3	0.24

By **Table 4.2.4**, we have $d_{pv}(P_3,S) < d_{pv}(P_1,S) < d_{pv}(P_2,S)$. The result is same as the result in [6,15,19]. Also, we have the DC in **Table 4.2.4 ,** is greater than the result of the specific distance metric. So, we use the Proposed new distance measure d_{pv} can be more accurate for pattern recognition.

Example 4.3 [6] The patterns P_1, P_2, P_3 and the test sample S, as presented in the following Table 4.3.1

Table 4.3.1

	A_1	A_2	A_3
P_1	(0.15,0.25)	(0.25,0.35)	(0.35,0.45)
P_2	(0.05,0.51)	(0.15,0.25)	(0.25,0.35)
P_3	(0.16,0.26)	(0.26,0.36)	(0.36,0.46)
S	(0.30,0.20)	(0.40,0.30)	(0.50,0.40)

 https://doi.org/10.37896/pd91.5/91513

In **Table 4.3.2**, we summarize the proposed new distance measures and results using **Algorithm 4.1**.

Table 4.3.2

Distance	**distance measures**			**Result**	
	$d_{pv}(P_1,S)$	$d_{pv}(P_2,S)$	$d_{pv}(P_3,S)$	**CS**	**DC**
d_{pv}	0.1563	0.2938	0.35	P_1	0.33

In **Table 4.3.3**, we summarize the results of different distance measures and pattern recognition.

Table 4.3.3

Distances	**distance measures**			**Result**	
	$d(P_1,S)$	$d(P_2,S)$	$d(P_3,S)$	**CCS**	**DC**
d_T	0.05	0.12	0.05	-	-
d_R	0.04	0.07	0.04	-	-
d_L	3.70×10^{-17}	3.70×10^{-17}	3.70×10^{-17}	-	-
d_{KD}	0.10	0.15	0.10	-	-
d_M	0.10	0.15	0.10	-	-
d_{LA}	0.07	0.08	0.07	-	-
d_G	0.05	0.08	0.05	-	-
d_H	0.15	0.30	0.14	P_3	0.17
d_E	0.13	0.28	0.12	P_3	0.17
d_{nH}	0.15	0.25	0.14	-	-
d_l	0.15	0.30	0.14	P_3	0.17
d_1	0.10	0.20	0.09	P_3	0.12
d_2^1	0.10	0.15	0.10	-	-
d_p	0.15	0.30	0.14	P_3	0.17
d_s	0.11	0.11	0.06	-	-
d_h	0.14	0.19	0.10	P_3	0.13
d_f	0.20	0.40	0.19	P_3	0.22
d_{pv}	0.16	0.29	0.35	P_1	0.33

distance measure d_{pv} has a higher confident measure than the other distance measures referenced in **Table 4.3.3**. This shows that the distance measure d_{pv} can be more accurate for pattern recognition.

Example 4.4 [35] Suppose that there are three patients: P_1, P_2, P_3 , i.e., $P=\{P_1, P_2, P_3\}$. The set of symptoms $S=\{S_1, S_2, S_3, S_4, S_5\}$. The set of diseases $D=\{D_1, D_2, D_3\}$. We have provided intuitionistic fuzzy relation $P\rightarrow S$ and $S\rightarrow D$ in **Tables 4.4.1** and **Table 4.4.2**. In **Table 4.4.5**, the proposed new distance measures d_{pv} between patients and diagnoses are presented.

 https://doi.org/10.37896/pd91.5/91513

Step 1

Table 4.4.1 Symptoms characteristic for the patients

Q	S_1	S_2	S_3	S_4	S_5
P_1	(0.7,0.2)	(0.6,0.2)	(0.3,0.7)	(0.5,0.2)	(0.2,0.7)
P_2	(0.7,0.1)	(0.8,0.2)	(0.1,0.6)	(0.2,0.7)	(0.1,0.5)
P_3	(0.5,0.1)	(0.5,0.3)	(0.3,0.5)	(0.7,0.1)	(0.3,0.5)

Table 4.4.2 Symptoms characteristic for the diagnoses

R	S_1	S_2	S_3	S_4	S_5
D_1	(0.4,0.1)	(0.3,0.5)	(0.1,0.6)	(0.4,0.3)	(0.1,0.6)
D_2	(0.5,0.1)	(0.3,0.6)	(0.1,0.9)	(0.7,0.1)	(0.2,0.8)
D_3	(0.6,0.3)	(0.6,0.2)	(0.2,0.7)	(0.2,0.7)	(0.1,0.8)

Step 2

Let us consider **Table 4.4.1**, each Symptoms characteristic for the patients are described by three numbers μ, ν and π in **Table 4.4.3**. Similarly, for **Table 4.4.2**, each Symptoms characteristic for the diagnoses are described by three numbers μ, ν and π in **Table 4.4.4**.

Table 4.4.3 Symptoms characteristic for the patients

Q	S_1	S_2	S_3	S_4	S_5
P_1	(0.7,0.2,0.1)	(0.6,0.2,0.2)	(0.3,0.7,0.0)	(0.5,0.2,0.3)	(0.2,0.7,0.1)
P_2	(0.7,0.1,0.2)	(0.8,0.2,0.0)	(0.1,0.6,0.3)	(0.2,0.7,0.1)	(0.1,0.5,0.4)
P_3	(0.5,0.1,0.4)	(0.5,0.3,0.2)	(0.3,0.5,0.2)	(0.7,0.1,0.2)	(0.3,0.5,0.2)

Table 4.4.4 Symptoms characteristic for the diagnoses

R	S_1	S_2	S_3	S_4	S_5
D_1	(0.4,0.1,0.5)	(0.3,0.5,0.2)	(0.1,0.6,0.3)	(0.4,0.3,0.3)	(0.1,0.6,0.3)
D_2	(0.5,0.1,0.4)	(0.3,0.6,0.1)	(0.1,0.9,0.0)	(0.7,0.1,0.2)	(0.2,0.8,0.0)
D_3	(0.6,0.3,0.1)	(0.6,0.2,0.2)	(0.2,0.7,0.1)	(0.2,0.7,0.1)	(0.1,0.8,0.1)

Step 3

Let us consider **Table 4.4.3** where the element $P_1 \rightarrow S_k$ with **Table 4.4.4** where the element $S_k \rightarrow D_1$. The proposed new distance measure $d_{pv}(P_1, D_1)$between $P_1 \rightarrow S_k$and $S_k \rightarrow D_1$ is obtained by Definition 3.1,

$$d_{pv}(P_1, D_1) = \frac{1}{4}\int_0^1 \sum_{i=1}^{5} \{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|\, \lambda + |\pi_A(x_i) - \pi_B(x_i)|\lambda^2\}\, d\lambda$$

 https://doi.org/10.37896/pd91.5/91513

$$= \frac{1}{4}\int_0^1 [\{|\,0.7-0.4| + |\,0.2-0.1|\,\lambda + |0.1-0.5|\lambda^2\} \\ + \{|\,0.6-0.3| + |\,0.2-0.5|\,\lambda + |0.2-0.2|\lambda^2\} \\ + \{|\,0.3-0.1| + |\,0.7-0.6|\,\lambda + |0.0-0.3|\lambda^2\} \\ + \{|\,0.5-0.4| + |\,0.2-0.3|\,\lambda + |0.3-0.3|\lambda^2\} \\ + \{|\,0.2-0.1| + |\,0.7-0.6|\,\lambda + |0.1-0.3|\lambda^2\}]\,d\lambda$$

$$= \frac{1}{4}\int_0^1 [\{0.3 + 0.1\lambda + 0.4\lambda^2\} + \{0.3 + 0.3\,\lambda + 0.0\lambda^2\} \\ + \{0.2 + 0.1\,\lambda + 0.3\lambda^2\} + \{0.1 + 0.1\,\lambda + 0.0\lambda^2\} \\ + \{0.1 + 0.1\,\lambda + 0.2\lambda^2\}]\,d\lambda$$

$$= \frac{1}{4}\int_0^1 [1 + 0.7\lambda + 0.9\lambda^2]\,d\lambda$$

$$=0.4125$$

Similarly, the remaining distance measures are calculated as:
$d_{pv}(P_1, D_2)$ =0.3875,$d_{pv}(P_1, D_3) = 0.2625$, $d_{pv}(P_2, D_1)$ =0.4167, $d_{pv}(P_2, D_2)$ =0.6167, $d_{pv}(P_2, D_3) = 0.2417$,$d_{pv}(P_3, D_1)$ =0.3708, $d_{pv}(P_3, D_2) =$ 0.2917, $d_{pv}(P_3, D_3)$ =0.475. We put all $d_{pv}(P_j, D_t)$in **Table 4.4.5** .

Step 4

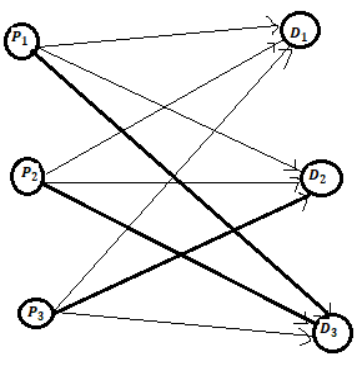
In following **Table 4.4.5**, the proposed new distance measures d_{pv}between patients and diagnoses are presented.

Table 4.4.5

$d_{pv}(P_j, D_t)$	D_1	D_2	D_3
P_1	0.4125	0.3875	**0.2625**
P_2	0.4167	0.6167	**0.2417**
P_3	0.3583	**0.2917**	0.475

The above **Table 4.4.5,** the proposed new distance measures degree d_{pv}between patients and diagnoses can be represented in the form of a graph namely network as follows:

Figure 1: Fuzzy medical diagnosis network



In the above network, nodes or vertices denote the patients and diseases, lengths or edges denote the assumption of diseases to the patients. The darken edges denotes the strong confirmation of disease to the patient.

ISSN: 0369-8963

According to the principle of minimum distance degree, the lower proposed new distance measures degree indicates a proper diagnosis. **Table 4.4.5** shows that P_1suffers from D_3, P_2suffers from D_3, P_3 suffers from D_2**.**

Example 4.5 [26,9,29,39,33,32,6] Suppose that there are four patients Al, Bob, Joe, Ted, represented as P = {Al, Bob, Joe, Ted}.
Their symptoms are $s = \{Temperature, Headache, Stomach\ pain, Cough, Chest\ pain\}$ The set of diagnoses is defined as D = {Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}. **Table 4.5.1** characterizes the intuitionistic fuzzy relation P →S. **Table 4.5.2** describes the intuitionistic fuzzy relation S → D. Each element of the tables is described by IFMS, which is a pair of numbers corresponding to the membership and non-membership values, respectively.

Step 1

Table 4.5.1 Symptoms characteristic for the patients

Q	**Temperature**	**Headache**	**Stomach pain**	**Cough**	**Chest pain**
Al	(0.8,0.1)	(0.6,0.1)	(0.2,0.8)	(0.6,0.1)	(0.1,0.6)
Bob	(0.0,0.8)	(0.4,0.4)	(0.6,0.1)	(0.1,0.7)	(0.1,0.8)
Joe	(0.8,0.1)	(0.8,0.1)	(0.0,0.6)	(0.2,0.7)	(0.0,0.5)
Ted	(0.6,0.1)	(0.5,0.4)	(0.3,0.4)	(0.7,0.2)	(0.3,0.4)

Table 4.5.2 Symptoms characteristic for the diagnoses

R	**Temperature**	**Headache**	**Stomach pain**	**Cough**	**Chest pain**
Viral Fever	(0.4, 0.0)	(0.3, 0.5)	(0.1, 0.7)	(0.4, 0.3)	(0.1, 0.7)
Malaria	(0.7,0.0)	(0.2,0.6)	(0.0,0.9)	(0.7,0.0)	(0.1,0.8)
Typhoid	(0.3,0.3)	(0.6,0.1)	(0.2,0.7)	(0.2,0.6)	(0.1,0.9)
Stomach problem	(0.1,0.7)	(0.2,0.4)	(0.8,0.0)	(0.2,0.7)	(0.2,0.7)
Chest problem	(0.1,0.8)	(0.0,0.8)	(0.2,0.8)	(0.2,0.8)	(0.8,0.1)

Step 2

Let us consider **Table 4.5.1**, each Symptoms characteristic for the patients are described by three numbers μ, ν and π in **Table 4.5.3**. Similarly, for **Table 4.5.2**, each Symptoms characteristic for the diagnoses are described by three numbers μ, ν and π in **Table 4.5.4**.

Table 4.5.3 Symptoms characteristic for the patients

Q	**Temperature**	**Headache**	**Stomach pain**	**Cough**	**Chest pain**
$\boldsymbol{Al}$	(0.8,0.1,0.1)	(0.6,0.1,0.3)	(0.2,0.8,0.0)	(0.6,0.1,0.3)	(0.1,0.6,0.3)
$\boldsymbol{Bob}$	(0.0,0.8,0.2)	(0.4,0.4,0.2)	(0.6,0.1,0.3)	(0.1,0.7,0.2)	(0.1,0.8,0.1)
$\boldsymbol{Joe}$	(0.8,0.1,0.1)	(0.8,0.1,0.1)	(0.0,0.6,0.4)	(0.2,0.7,0.1)	(0.0,0.5,0.5)
$\boldsymbol{Ted}$	(0.6,0.1,0.3)	(0.5,0.4,0.1)	(0.3,0.4,0.3)	(0.7,0.2,0.1)	(0.3,0.4,0.3)

 https://doi.org/10.37896/pd91.5/91513

ISSN: 0369-8963

Table 4.5.4 Symptoms characteristic for the diagnoses

R	Temperature	Headache	Stomach pain	Cough	Chest pain
Viral Fever	(0.4, 0.0,0.6)	(0.3, 0.5,0.2)	(0.1, 0.7,0.2)	(0.4, 0.3,0.3)	(0.1, 0.7,0.2)
Malaria	(0.7,0.0,0.3)	(0.2,0.6,0.2)	(0.0,0.9,0.1)	(0.7,0.0,0.3)	(0.1,0.8,0.1)
Typhoid	(0.3,0.3,0.4)	(0.6,0.1,0.3)	(0.2,0.7,0.1)	(0.2,0.6,0.2)	(0.1,0.9,0.0)
Stomach problem	(0.1,0.7,0.2)	(0.2,0.4,0.4)	(0.8,0.0,0.2)	(0.2,0.7,0.1)	(0.2,0.7,0.1)
Chest problem	(0.1,0.8,0.1)	(0.0,0.8,0.2)	(0.2,0.8,0.0)	(0.2,0.8,0.0)	(0.8,0.1,0.1)

Step 3

Let us consider **Table 4.5.3** where the element $P_j \rightarrow S_k$ with **Table 4.5.4** where the element $S_k \rightarrow D_t$. The proposed new distance measures $d_{pv}(P_j, D_{1t})$between $P_j \rightarrow S_k$and $S_k \rightarrow D_t$ is obtained by Definition 3.1, are as follows

$d_{pv}(P_1, D_1) = 0.44$, $d_{pv}(P_1, D_2) = 0.28$, $d_{pv}(P_1, D_3) = 0.43$,$d_{pv}(P_1, D_4) = 0.35$, $d_{pv}(P_1, D_5) = 0.98$

$d_{pv}(P_2, D_1)$ =0.63, $d_{pv}(P_2, D_2) = 0.88$, $d_{pv}(P_2, D_3) = 0.5$,$d_{pv}(P_2, D_4) = 0.25$, $d_{pv}(P_2, D_5) = 0.71$

$d_{pv}(P_3, D_1)$ =0.65, $d_{pv}(P_3, D_2)$ =0.75, $d_{pv}(P_3, D_3) = 0.48$,$d_{pv}(P_3, D_4) = 0.88$, $d_{pv}(P_3, D_5) = 0.97$

$d_{pv}(P_4, D_1)$ =0.45, $d_{pv}(P_4, D_2)$ =0.45, $d_{pv}(P_4, D_3) = 0.54$,$d_{pv}(P_4, D_4) = 0.72$, $d_{pv}(P_4, D_5) = 0.9$

We put all $d_{pv}\left(P_j, D_t\right)$in **Table 4.5.5**.

Step 4

In following **Table 4.5.5,** the proposed new distance measures degree d_{pv}between patients and diagnoses are presented.

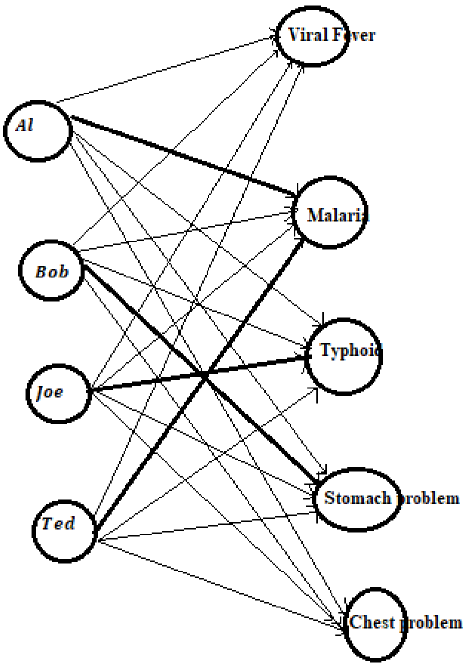
Table 4.5.5

$d_{pv}(P_j, D_t)$	Viral Fever	Malaria	Typhoid	Stomach problem	Chest problem
Al	0.44	**0.28**	0.43	0.35	0.98
Bob	0.63	0.88	0.5	**0.25**	0.71
Joe	0.65	0.75	**0.48**	0.88	0.97
Ted	0.45	**0.44**	0.54	0.72	0.9

The above **Table 4.5.5,** the proposed new distance measures degree d_{pv}between patients and diagnoses can be represented in the form of a graph namely network as follows:

 https://doi.org/10.37896/pd91.5/91513

Figure 2: Fuzzy medical diagnosis network



In the above network, nodes or vertices denote the patients and diseases, lengths or edges denote the assumption of diseases to the patients. The darken edges denotes the strong confirmation of disease to the patient.

According to the principle of lowest distance degree, the minimum distance degree indicates a proper diagnosis. **Table 4.5.5** shows that Al suffers from Malaria, Bob suffers from Stomach problem, Joe suffers from Typhoid, and Ted suffers from Malaria.
In **Table 4.5.5**, the diagnosis results for this case obtained in previous study and this study have been presented.

Table 4.5.5

The result in	Al	Bob	Joe	Ted
[22]	Malaria	Stomach problem	Typhoid	Malaria
[24]	Malaria	Stomach problem	Malaria	Malaria
[21]	Malaria	Stomach problem	Typhoid	Viral Fever
[23]	Malaria	Stomach problem	Typhoid	Viral Fever
[7]	Viral	Stomach problem	Typhoid	Malaria
[12]	Fever	Stomach problem	Typhoid	Viral Fever
[16]	Malaria	Stomach problem	Typhoid	Malaria
New	Malaria	Stomach problem	Typhoid	Malaria
proposed	Malaria			

 https://doi.org/10.37896/pd91.5/91513

Conclusion

Although many distance measures between IFS have been proposed intuitionistic fuzzy multisets has not considered so far. In this paper, we introduced the new distance measure between intuitionistic fuzzy multisets and applied it to pattern recognition and medical diagnosis problems which are already applied with various distance measures. The results are compared with the other distance measures results.

References

[1] Zadeh LA. Fuzzy sets. Inf Control 1965;8:338–353.
[2] Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets Syst 1986;20:87–96.
[3] Ronald R.Yager, "On the Theory of Bags", International Journal of General Systems, 1986, Vol.13, pp.23 – 37.
[4] Shinoj T.K, Sunil Jacob John. Intuitionistic fuzzy Multisets and its application in medical diadnosis.International Journal of Mathematical,Computational,Physical, Electrical and Engginering 2012, vol: 6,No:1
[5] Eulalia Szmidt and Janusz Kacprzyk, "On measuring distances between Intuitionistic Fuzzy Sets", NIFS, Vol. 3, No.4, 1 – 13, 1997.
[6] Szmidt E, Kacprzyk J. Distances between intuitionistic fuzzy sets. Fuzzy Sets Syst 2000;114:505–18.
[7] Szmidt E, Kacprzyk J. A similarity measure for intuitionistic fuzzy sets and its application in supporting medical diagnostic reasoning. artificial intelligence and soft computing - ICAISC 2004. 2004. p. 388–93.
[8] Li DF, Cheng CT. New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions. Pattern Recognit Lett 2002;23:221–5.
[9] Wang W, Xin X. Distance measure between intuitionistic fuzzy sets. Pattern Recognit Lett 2005;26:2063–9.
[10] Jin Han Park, Ki Moon Lim, Jong Seo Park and Young Chel Kwun, "Distances between Interval-valued Intuitionistic Fuzzy Sets", Journal of Physics, Conference series 96, 2008, 012089
[11] Supriya Kumar De, Ranjit Biswas, Akhil Ranjan, " An application of Intuitionistic Fuzzy Sets in Medical Diagnosis", Fuzzy Sets and Systems, 117 (2001), 209-213.
[12] Szmidt E, Kacprzyk J "Intuitionistic Fuzzy Sets in Intelligent Data Analysis for Medical Diagnosis", Computational Sciences, ICCS 2001, pp 263 – 271.
[13] Hatzimichailidis AG, Papakostas GA, Kaburlasos VG. A novel distance measure of Intuitionistic fuzzy sets and its application to pattern recognition problems. Int J Intell Syst 2012;27:396–409.
[14] P. A. Ejegwa, A. J. Akubo O. M. Joshua, "Intuitionistic Fuzzy Set and its Application in Career Determination via Normalized Euclidean Distance Method" , European Scientific Journal May 2014, Vol 10, No.15, page 529–536.
[15] Atanassov KT. New operations defined over the intuitionistic fuzzy sets. Fuzzy Sets Syst 1994;61:137–142.
[16] Minxia Luo, Ruirui Zhao, A distance measure between intuitionistic fuzzy sets and its application in medical diagnosis,Artificial Intelligence In Medicine,2018.
[17] W G, Cheng QX, Zhang DZ. Basis of real variable function and functional analysis. Higher Education Press; 1983.

[18] Grzegorzewski P. Distances between intuitionistic fuzzy sets and interval-valued fuzzy sets based on the hausdorff metric. Fuzzy Sets Syst 2004;148:319–28.
[19] Yang Y, Chiclana F. Consistency of 2D and 3D distances of intuitionistic fuzzy sets. Expert Syst Appl 2012;39:8665–70.
[20] Sharma PK, Habib SA, Yamin M. Intuitionistic fuzzy database for medical diagnosis. International conference on computing for sustainable global development 2016, pp 1408–11.
[21] Pramanik S, Mondal K. Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-attribute decision making. Glob J Adv Res 2015;2:464–71.
[22] Own CM. Switching between type-2 fuzzy sets and intuitionistic fuzzy sets: an application in medical diagnosis. Appl Intell 2009;31:283–91.
[23] Wei CP, Wang P, Zhang YZ. Entropy, similarity measure of interval-valued intuitionistic fuzzy sets and their applications. Inf Sci 2011;181:4273–86.
[24] De SK, Biswas R, Roy AR. An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets Syst 2001;117:209–213.