Fuzzy Z- Number Shortest Path Problem using Dijkstra Algorithm

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Abstract

The fuzzy shortest path problem is one of the most researched topics when it comes to graph problems with uncertainty.We discuss the problem of finding the fuzzy z-number shortest paths from a fixed origin to a specified nodes in a network with the arc length of the network takes fuzzy z- number, instead of real numbers, namely, fuzzy z-number. Based on Dijkstra's approach, an algorithm is suggested for solving the fuzzy z-number shortest path problem.

Key words

fuzzy set, z-number, fuzzy z-number, fuzzy z-graph fuzzy z-number Dijkstra's, shortest path problem.

Introduction

L.A.Zadeh [13] introduced the concept of z-numbers in 2011. Floyd [1], R. W, S. Chanas and J. Kamburowski [2] and C. M. Klein [3] have applied the cocept of the fuzzy shortest route problem . S. Okada and T. Soper, S. Moazeni and many others have investigated a shortest path problem on a network with fuzzy arc lengths in [4] ,and [5] .In [10] T. N. Chuang and J. Y. Kung has discussed the fuzzy shortest path length and the corresponding shortest path in a network,2005. J.S. Yao and K.M. Wu. [11] has concentrated on Ranking Fuzzy numbers based on decomposition principle and signed distance in 2000. Azriel Rosenfeld introduced fuzzy graph in 1975. [14] and [15] Ronald R. Yager and M Shahila Bhanu, G Velammal have discussed the concept of Zadeh Z-numbers and its operations. In [16] and [17], Wen Jiang, ChunheXie, Yu Luo ,Youngchuan Tang and Matteo Brnelli have concentrated on ranking z-numbers and different ranking methods for fuzzy numbers .In [18] Siddhartha sankar biswas has applied the concepts of Z-Dijkstra's Algorithm to solve Shortest Path Problem in z-Graph .

preliminaries

In this section, we recall some relevant definitions and results. Throughout this paper, X is a universal set.

Definition 2.1 A fuzzy subset A in a universe X may be given as $A = \{\langle x, \mu_A(x) \rangle / x \in X\}$, Where $\mu_A: X \to [0,1]$ is called the membership function of A and $\mu_A(x)$ is the degree of membership value to which x in A.

Definition 2.2

A fuzzy set A defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics

a) $\mu_A: X \to [0,1]$ is continuous,
b) $\mu_A(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
c) μ_A (x) is strictly increasing on $[a , b]$ and strictly decreasing on $[c , d]$
d) $\mu_A(x) = 1$ for all $x \in [b, c]$, where $a < b < c < d$

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Definition 2.3

The membership function of the triangular fuzzy number $A(a,b,c)$ is defined by $\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & if\ x \epsilon [a,b] \\ 1, & if\ x = b \\ \frac{c-x}{c-b}, & if\ x \epsilon [b,c] \\ 0, & otherwise \end{cases}$

Definition 2.4

Let F be a set of fuzzy numbers. A ranking function r_k on F is a real valued function defined on F. Given a ranking function $r_k$on F we can order the elements in F as follows: $A_1 \leq A_2$ if and only if $r_k(A_1) \leq r_k(A_2)$.

Definition 2.5

Many types of ranking functions are widely used namely Centre of Gravity, Centre of Maxima, Median and Yager's. In this paper we are using are using ranking from Median method. Consider the triangular fuzzy number C(a,b,c),Median method for ranking is defined by $r(C) = \frac{a+b+c}{3}$.

Definition 2.6

A z-number is an ordered pair of fuzzy numbers. Indicated as z = (A, B). The first component A is a restriction of real-valued uncertain variable X. The subsequent part, B, is a proportion of dependability (certainty) of the main part. A z-number, gives data about the unsure variable, yet in addition the unwavering quality of the data.

Definition 2.7

If $z = (a,b,c)$ is a given triangular fuzzy number the rank of A, $r_k(A) = \frac{a+b+c}{3}$. Define max (A,B) = A, and min (A,B) = B, if $r_k(A) \geq r_k(B)$.

Definition 2.8

Let r be a ranking function of fuzzy numbers. Let $z_1 = (A_1, B_1)$ and $z_2 = (A_2, B_2)$ be any two z–numbers. Define $z_1 \leq z_2$ iff $r(A_1) \leq r(A_2)$ and in that case, define max (z_1, z_2) = z_2 iff $z_1 \leq z_2$ and min(z_1, z_2) = z_1 iff $z_1 \leq z_2$

Definition 2.9

Let $z_1 = (A_1, B_1)$ and $z_2 = (A_2, B_2)$ are two triangular z-number, where A_1= (a_1, a_2, a_3), B_1 = (b_1, b_2, b_3), A_2= (c_1, c_2, c_3), B_1= (d_1, d_2, d_3) are triangular fuzzy numbers.

Definition 2.10

Addition of two triangular Z-numbers is

z_1 (+, max) z_2 = $((a_1 + c_1, a_2 + c_2, a_3 + c_3), B_2)$ if $r_k(B_1) \leq r_k(B_2)$ (or)

z_1 (+, max) z_2 = $((a_1 + c_1, a_2 + c_2, a_3 + c_3), B_1)$ if $r_k(B_1) \geq r_k(B_2)$

Definition 2.11

Subtraction of two triangular Z-numbers is

z_1 (−, min) z_2 = $((a_1 - c_3, a_2 - c_2, a_3 - c_1), B_2)$ if $r_k(B_1) \leq r_k(B_2)$ (or)

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z_1 $(-$, min$)$ z_2 = $((a_1 - c_3, a_2 - c_2, a_3 - c_1),\ B_1)$ if $r_k(B_1) \ \geq \ r_k(B_2)$

Fuzzy z- number

Definition 3.1

Consider a path P from vertex u to vertex v. Suppose it consists of edges e_1, e_2,….. e_n. Suppose the weights of these edges are the fuzzy z-numbers (A_1, B_1), (A_2, B_2),……..,(A_n, B_n).The fuzzy z- number length of the path P is defined by z^P = (A_1, B_1)(+,min)(A_2, B_2)……............(+,min)(A_n, B_n) and it is denoted by z^P. Here our aim is to find the path between u and v whose fuzzy z- number length is minimum.(i.e.,z-SP between u & v) and the minimum fuzzy z-number length of the path is denoted by z_{SD}^P.

Definition 3.2

Let P and Q consists of edges of the given network diagram. If we use $z^{\Cap}$ (r_1, r_2) with r_1 = r_2 = r we get $z^Q <\ z^P$.On the other hand, If we use $z^{\Cup}$ (r_1, r_2) with r_1= r_2 = r we get $z^P<\ z^Q$.If $(a, b, c)$is a given triangular fuzzy z-number the rank of A, $r_k(A) = \frac{a+b+c}{3}$. Define max(A,B) = A and min (A,B) = B, if $r_k(A) \ \geq \ r_k(B)$.

Dijkstra algorithm

We have described the proposed Dijkstra algorithm for finding z-number fuzzy shortest path and duration are given below.

Step 1 Label the source edge with permanent label [((0,0,0),(1,1,1)) , $-$].

Step 2 (i) Compute the temporary label $[z^P, i]$ for each edge j that can be reached from i, provided j is not permanently labeled.

(ii) If node j is already labeled as $[z^P, k]$ through another edge k and if $z^{\Cup}\ (r, i) <\ z^{\Cup}\ (r, k)$ replace $[z^P, k]$ with $[z^P, i]$.

Step 3 If all the edges are permanently labeled, the algorithm terminates.

Step 4 Otherwise , choose the label $[z^P$, s] with shortest distance (z^P) from the list of temporary labels. Set $i = \ r$ and repeat step 2.

Step 5 Obtain the shortest path between edge 1 and the destination edge j by tracing backward through the network using the label’s information.

Step 6 The path having the minimum value and is identified as the z-number fuzzy shortest path and the corresponding path length . The fuzzy z-number shortest length of the path is denoted by z_{SD}^P.

Example 3.4

The following is an example of fuzzy z-number graph. Obtain the fuzzy z-number shortest length of the path between edges using Dijktra’s algorithm.

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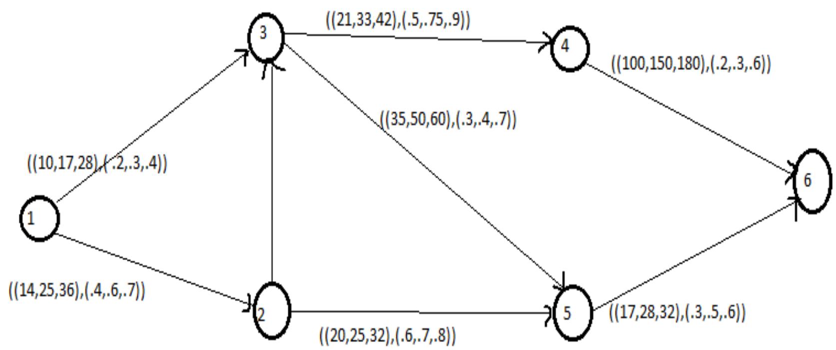
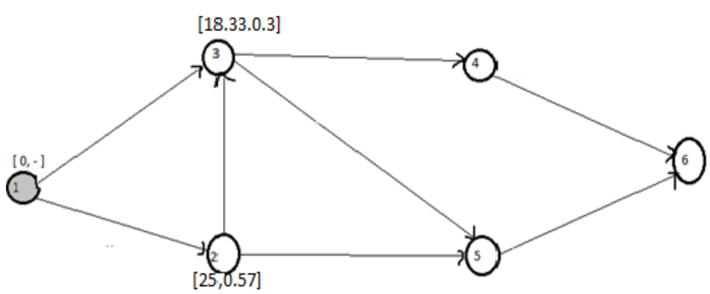
Consider the z-number fuzzy shortest path between edges e_1 to e_6 .



Iteration 0

Assign the permanent label [((0,0,0),(1,1,1)) , −] to e_1.



Iteration 1

Edge e_2 and e_3 can be reached from e_1.For e_2, z-number fuzzy path P consists of edges $e_1 \rightarrow e_2$ and z-number fuzzy path Q consists of edges $e_1 \rightarrow e_3$

Table 1: Results of the network based on z-number fuzzy graph have been summarized .

Table -1

Edges	Label	$r = \frac{a+b+c}{3}$		z^r	Status
		$r(A_i)$	$r(B_i)$		
e_1	[((0,0,0),(1,1,1)) , −]	0	1	-	P
e_2	[((14,25,36),(0.4,0.6,0.7)) , e_1]	25	0.57	2	T
e_3	[((10,17,28),(0.2,0.3,0.4)) , e_1]	18.33	0.3	1	T

With respect to $z^P < z^Q$, the minimum is 18.33.Thus the status of edge e_3 is changed to permanent.

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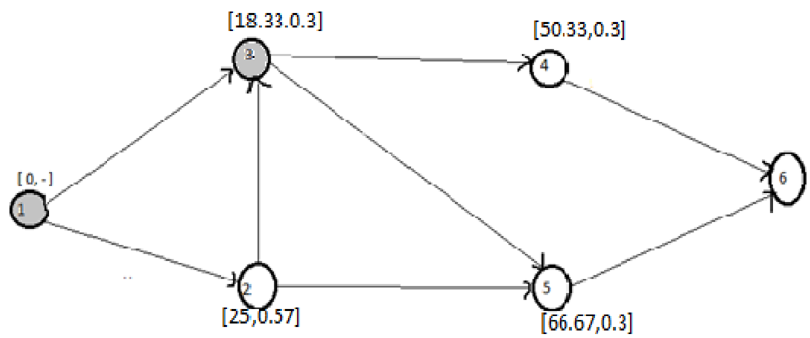


Iteration 2

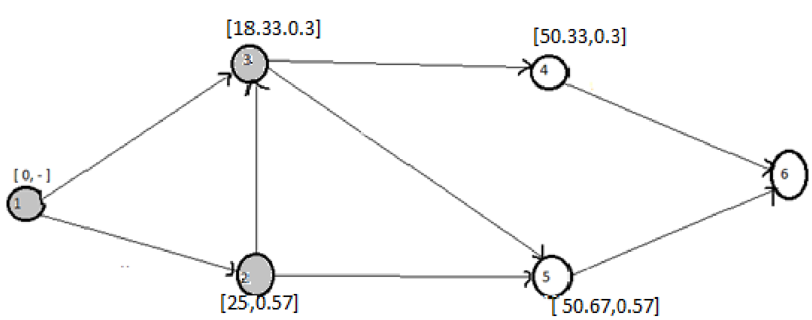
Edge e_4 and e_5 can be reached from e_3. For e_4, z-number fuzzy path P consists of edges $e_1 \rightarrow e_3$ and $e_3 \rightarrow e_4$

Table 2: Results of the network based on z-number fuzzy graph have been summarized .

Table - 2

Edges	Label	$r = \frac{x+y+z}{3}$		z^r	Status
		$r(A_i)$	$r(B_i)$		
e_1	$[((0,0,0),(1,1,1)), -]$	–	–	–	P
e_2	$[((14,25,36),(0.4,0.6,0.7)), e_1]$	25	0.57	1	T
e_3	$[((10,17,28),(0.2,0.3,0.4)), e_1]$	–	–	–	P
e_4	$[((31,50,70),(0.2,0.3,0.4)), e_3]$	50.33	0.3	2	T
e_5	$[((45,67,88),(0.2,0.3,0.4)), e_3]$	66.67	0.3	3	T

With respect to $z^P < z^Q$, the minimum is 25.Thus the status of edge e_2 is changed to permanent.



Iteration 3

Edge e_3 and e_5 can be reached from e_2. Thus the status of edge e_3 is already permanent. For e_5, z-number fuzzy path P consists of edges $e_1 \rightarrow e_2$ and $e_2 \rightarrow e_5$.

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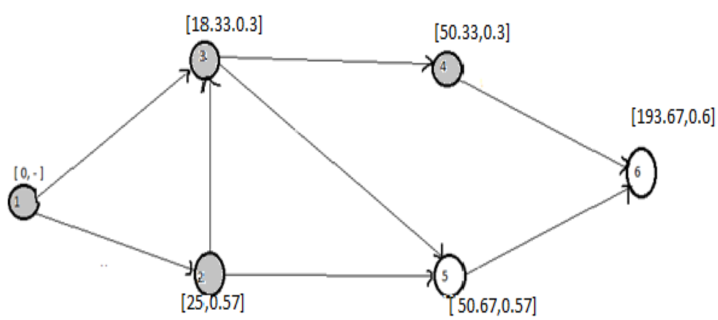
Table 3: Results of the network based on z-number fuzzy graph have been summarized .

Table - 3

Edges	Label	$r = \frac{x+y+z}{3}$		z^r	Status
		$r(A_i)$	$r(B_i)$		
e_1	$[((0,0,0),(1,1,1)), -]$	$-$	$-$	$-$	P
e_2	$[((14,25,36),(0.4,0.6,0.7)), e_1]$	$-$	$-$	$-$	P
e_3	$[((10,17,28),(0.2,0.3,0.4)), e_1]$	$-$	$-$	$-$	P
e_4	$[((31,50,70),(0.2,0.3,0.4)), e_3]$	50.33	0.3	1	T
e_5	$[((34,50,68),(0.4,0.6,0.7)), e_2]$ (or) $[((45,67,88),(0.2,0.3,0.4)), e_3]$	50.67 (or) 66.67	0.57 (or) 0.3	2 (or) 3	T

With respect to $z^P < z^Q$ the minimum is 50.33. Thus the status of edge e_4 is changed to permanent.



Iteration 4

Edge e_6 can be reached from e_4. For e_6, Path P consists of edges $e_4 \rightarrow e_6$.

Table 4: Results of the network based on z-number fuzzy graph have been summarized .

Table - 4

Edges	Label	$r = \frac{x+y+z}{3}$		z^r	Status
		$r(A_i)$	$r(B_i)$		
e_1	$[((0,0,0),(1,1,1)), -]$	$-$	$-$	$-$	P
e_2	$[((14,25,36),(0.4,0.6,0.7)), e_1]$	$-$	$-$	$-$	P
e_3	$[((10,17,28),(0.2,0.3,0.4)), e_1]$	$-$	$-$	$-$	P
e_4	$[((31,50,70),(0.2,0.3,0.4)), e_3]$	$-$	$-$	$-$	P
e_5	$[((34,50,68),(0.4,0.6,0.7)), e_2]$	50.67	0.57	1	T
e_6	$[((131,200,250),(0.2,0.3,0.4)), e_4]$	193.67	0.3	2	T

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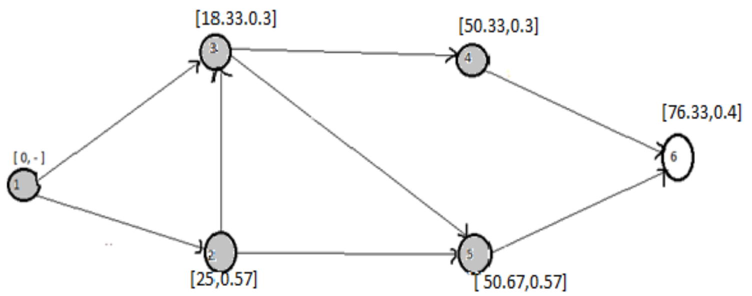
With respect to $z^P < z^Q$, the minimum is 50.67. Thus the status of edge e_5 is changed to permanent.



Iteration 5

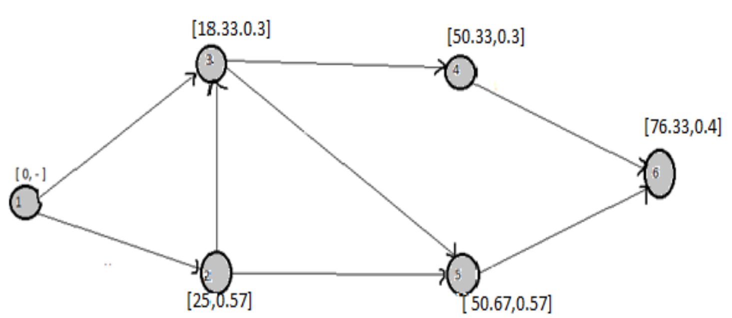
The only temporary edge e_6 can be reached from e_5. For e_6, z-number fuzzy path P consists of edges $e_5 \rightarrow e_6$.

Table 5: Results of the network based on z-number fuzzy graph have been summarized .

Table - 5

Edges	Label	$r = \frac{x+y+z}{3}$		z^r	Status
		$r(A_i)$	$r(B_i)$		
e_1	[((0,0,0),(1,1,1)) , −]	−	−	−	P
e_2	[((14,25,36),(0.4,0.6,0.7)) , e_1]	−	−	−	P
e_3	[((10,17,28),(0.2,0.3,0.4)) , e_1]	−	−	−	P
e_4	[((31,50,70),(0.2,0.3,0.4)) , e_3]	−	−	−	P
e_5	[((34,50,68),(0.4,0.6,0.7)) , e_2]	−	−	−	P
e_6	[((51,78,100),(0.3,0.5,0.6)) ,e_5]	76.33	0.47	1	T

Thus the status of edge e_6 is changed to permanent.



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Iteration 6

The only temporary edge e_6 does not lead to any other edge, its status converted into permanent and the process ends.

Table 6: Results of the network based on z-number fuzzy graph have been summarized .

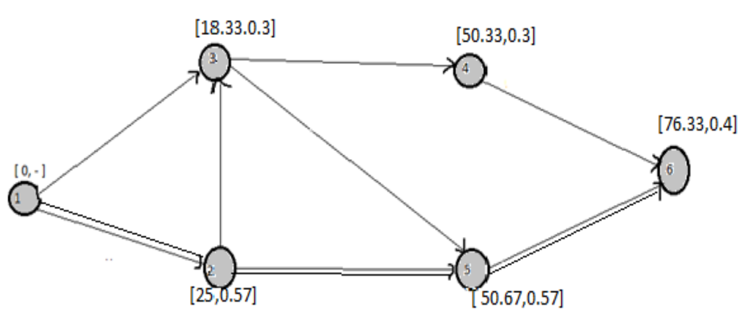
Table - 6

Edges	Label	Status
e_1	[((0,0,0),(1,1,1)) , −]	P
e_2	[((14,25,36),(0.4,0.6,0.7)) , e_1]	P
e_3	[((10,17,28),(0.2,0.3,0.4)) , e_1]	P
e_4	[((31,50,70),(0.2,0.3,0.4)) , e_3]	P
e_5	[((34,50,68),(0.4,0.6,0.7)) , e_2]	P
e_6	[((51,78,100),(0.3,0.5,0.6)) ,e_5]	P

Based on the **step 5**, the following sequence determines the fuzzy z-number shortest path from edge e_1 to e_6 is,

(e_6) →**[((51,78,100),(0.3,0.5,0.6)) ,e_5**] → (e_5) → **[((34,50,68),(0.4,0.6,0.7)),e_2]**→ (e_2) →**[((14,25,36),(0.4,0.6,0.7)),e_1]** → (e_1)

Thus, the required fuzzy z-number shortest path, z^P is $(e_1) \rightarrow (e_2) \rightarrow (e_5) \rightarrow (e_6)$



Hence the fuzzy z-number shortest length of the path is z_{SD}^P= ((51,78,100),(0.3,0.5,0.6))

Conclusion

In this paper, a fuzzy z-number shortest path length is obtained using a procedure based on Dijkstra algorithm in a fuzzy z-number network. The effectiveness of the proposed algorithm is verified with an example.

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